Bifurcation analysis on full annular rub of a nonlinear rotor system

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In this paper, analytical and numerical studies are carried out on the full annular rub motions of a nonlinear Jeffcott rotor. Transition sets of the synchronous full annular rub are given with the help of averaging method and constraint bifurcation theory to discuss the effects of system parameters on jump phenomena. Routh–Hurwitz criteria are employed to analyze the stability of synchronous full annular rub solution and determine the boundaries of static and Hopf bifurcations. Finally, the response and onset condition of reverse dry whip are investigated numerically, and at the same time, the influences of rotor parameters and rotation speed on the characteristics of the rotor response are investigated.

nonlinear rotor dynamics, synchronous full annular rub, constraint bifurcation, stability of motion, reverse dry whip


1 Introduction

Rotating machinery is widely used in engineering. In order to increase the efficiency of rotating machinery, the clearance between rotor and stator has been designed to be smaller and smaller, however, which leads to a larger possibility of impact-rub. Rubbing may induce the rotor’s dynamic instability, blade break, and other serious accidents. Therefore, how to avoid rub and how to reduce the harm caused by rubs become an important problem in rotor dynamics.

Generally, rub which happens near the rotor’s operation speed, can be divided into two classes: full annular rub and partial rub. There exist two kinds of annular rub in the rotor-stator system: synchronous full annular rub and reverse dry whip. Synchronous full annular rub is an unbalance-excited motion with small danger and light contact. However, the second one which is self-excited with an amplitude far more than the clearance may easily lead to the machinery’s catastrophic failure.

Many researchers paid attention to the rotor-to-stator rubbing problem [1–6]. Analytical, numerical methods and experiments were employed to investigate the possible contact responses induced by rubbing and derive the relationship between the contact responses and the parameters of rotor systems. Dynamics of full annular rub including synchronous and reverse dry whip were investigated by Yu and Muszynska [7, 8] experimentally and analytically. The response and stability for both two cases were obtained, and the effect of parameters, such as mass unbalance, dry friction, rotor damping, and seal stiffness, on shifting from synchronous response to reverse rub were discussed. Zhang et al. [9–11] studied the SFARM (synchronous full annular rub motion) of a single disk rotor and a flexible rotor, respectively. Perturbation method was employed to solve the exact solution and its stability regions. With the assumption of no rotor imbalance and rigid stator, simulations and tests were performed by Bartha [12] to determine the influence

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of the radial impact velocity of the rotor on backward whirl. Neglecting the imbalance of the rotor, Black [13] gave the onset condition of the counter-whirl with and without slip. Crandall [14] did the same study and found that there was a good correspondence between the derived condition and the experimental one in the case of small ratios of rotor radius to radial clearance. Jiang and Ulbrich [15] investigated a Jeffcott rotor with cross-coupling stiffness coefficients. The stability conditions of the period solution were discussed and the transition between periodic and quasi-periodic rub responses as well as between the full annular rubs and the partial rubs were provided. Then they did another paper [16] for the onset condition of reverse dry whip, formulated new motion equations for the rotor/stator system, and derived the occurrence conditions by using a developed numerical algorithm based on the multiple scale method. The dry friction whirl of a rotor/stator model, considering both of the vibration of the rotor and the stator as well as the friction and the deformation at the contact surfaces, were studied analytically by Jiang et al. [17] to present the existence boundaries and the whirl frequencies of the dry friction backward whirl. It was found that there were two existence dry friction backward whirl regions. The whirl frequencies in the two existence regions were quite different and may jump between the lower and the higher values with the variation of the rotating speed. Ma et al. [18, 19] discussed the influence of damping, eccentricity and clearance on the critical rub speed, obtained the analytical expression of critical rotation speed of rub. Liu et al. [20] studied the Hopf bifurcation of a full annular rub of a Jeffcott rotor. Groll and Ewins [21] described the periodic response and its stability by using the multiple scale method based on the harmonic balance method. However, most of the research works were been done with the linear rotor system. Thus, it is significant to study the full annular rub of a nonlinear rotor system which is more universal in engineering. Furthermore, impact-rub of the rotor system is a typical non-smooth problem in reality. Corresponding to different constraints, the rubbing forces have different expressions with and without rubbing. Thus, it is meaningful to study this problem by the constraint bifurcation theory. However, few works have been done in this area.

In this paper, analytical and numerical studies are carried out on the full annular rub motions of a nonlinear Jeffcott rotor to provide a basis for the parametric design of the rotor system. Averaging method is used to find the synchronous response of governing equations in section 2. In section 3, transition set for the synchronous full annular rub is given with the help of constraint bifurcation theory and C-L method to discuss the influence of system parameters on jump phenomena. Then in section 4, the Routh–Hurwitz criteria are employed to analyze the stability of synchronous full annular rub solution and determine the Hopf bifurcation boundary. Finally, the response and onset condition of reverse dry whip are investigated numerically, and based on the results, the effects of rotor parameters and rotation speed on the characteristics of the rotor response are investigated.

2 Governing equations

A model of full annular rub of nonlinear Jeffcott rotor shown in Figure 1 is considered in this paper, whose motion equation of which can be expressed as

\[ \begin{align*}
mx'' + cx + kx + \alpha x \left( x^2 + y^2 \right) + P_x &= me\omega^2 \cos \omega t, \\
my'' + cy + \alpha y \left( x^2 + y^2 \right) + P_y &= me\omega^2 \sin \omega t,
\end{align*} \]

(1)

where the rubbing forces shown in Figure 2 are combined by linear contact force and Coulomb friction force [15]:

\[ \begin{bmatrix}
P_x \\
P_y
\end{bmatrix} = \begin{cases}
0, & r < r_0, \\
-k_b \left( \frac{r_0}{r} \right) \left[ x - \mu y \right], & r > r_0,
\end{cases} \]

(2)

where \( c \) is the system damping, \( \alpha \) is the nonlinearity of the rotor, \( e \) is rotor mass eccentricity, \( r_0 \) is the clearance between the rotor and the stator, \( r = \sqrt{x^2 + y^2} \) is the radius displacement of the rotor, \( \mu \) is the friction coefficient, and \( k_b \) is the contact stiffness.

Let \( X = x/e, \ Y = y/e \) and \( \tau = \omega_0 t \); eq. (1) can be further rewritten in the following non-dimensional form:

\[ \begin{align*}
X'' + \epsilon \gamma X' + X + \epsilon \beta \left( X^2 + Y^2 \right) + P_x &= \epsilon E \Omega^2 \cos \Omega \tau, \\
Y'' + \epsilon \gamma Y' + Y + \epsilon \beta \left( X^2 + Y^2 \right) + P_y &= \epsilon E \Omega^2 \sin \Omega \tau,
\end{align*} \]

(3)

where