Thickness-stretch vibration of a crystal plate carrying a micro-rod array

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We analyze thickness-stretch vibrations of a plate of hexagonal crystal carrying an array of micro-rods with their bottoms fixed to the top surface of the plate. The rods undergo longitudinal vibrations when the crystal plate is in thickness-stretch motion. The plate is modeled by the theory of anisotropic elasticity. The rods are modeled by the one-dimensional structural theory for extensional vibration of rods. A frequency equation is obtained and solved using perturbation method. The effect of the rod array on the resonant frequencies of the crystal plate is examined. The results are potentially useful for using thickness-stretch modes of crystal plates to measure the mechanical properties of microrod arrays.

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Thickness-shear (TSh) and thickness-stretch (TSt) vibration modes of quartz and other crystal plates have been widely used for quartz crystal resonators (QCRs), filters and other acoustic wave devices. These devices provide frequency standards or perform frequency operations for time-keeping, telecommunication, control, and other applications. During the last couple of decades, applications of QCRs as acoustic wave sensors have grown rapidly [1–4]. These sensors are based on the frequency effects on a QCR from various measurands including stresses, temperature change, the inertia and stiffness of additional surface mass layers, and the viscosity and density of a fluid in contact with the QCR, etc. For example, the effect of the inertia of a thin mass layer on a QCR has been used to make quartz crystal microbalances (QCMs) which have become a basic tool in analytical chemistry and in monitoring thin film deposition. QCMs loaded with cells or biological/chemical fluids constitute a big class of biosensors and chemical sensors. Recently, due to the extensive effort on micro- and nano-technologies, various micro- or nano-scale beam arrays can be made using different techniques [5–9]. These new structures have great potentials for new devices including dynamic tuning of surface wetting, dry adhesives that mimic geeko foot fibrillars, efficient microneedles in drug delivery, substrates for sensing cell response and MEMS actuators. There is a strong need to measure the geometric and physical parameters of these small beam arrays. In this paper we propose to use a TSt mode crystal resonator and extensional vibrations of microbeams to measure the properties of the microbeams which may be more suitably called microrods when they are in extensional motion. To demonstrate this idea, we construct a theoretical model of a plate of hexagonal crystals carrying an array of microrods with their bottoms fixed to the top surface of the plate (see Figure 1). When the crystal plate is in TSt motion, the rods undergo extensional vibrations. The axial forces at the bottoms of the rods exert a...
normal force on the plate surface, thereby affecting the resonant frequencies of the plate. For a typical crystal plate, its thickness is of the order of 1 mm. We consider small rods whose effect on the resonant frequencies of the plate is a small perturbation which we will determine by a theoretical analysis. We will show that, through the rod array-frequency effect, information about the rod array can be extracted.

1 Plate TS\textit{t} motion

The six-fold axis of the crystal plate is indicated by an arrow labeled with “P” in Figure 1. Since the structure of the elasticity tensor of hexagonal crystals and polarized ceramics (transversely isotropic) are the same, the analysis below is also valid for plates of polarized ceramics. We are considering a frequency effect which is mechanical in nature. To illustrate the main idea, an elastic analysis is sufficient. Therefore, for the crystal plate, we use the equations of anisotropic elasticity and neglect piezoelectric coupling which is necessary in an electrically forced vibration analysis. In free vibration frequency analysis, the piezoelectric stiffening effect causes a shift in the reference frequencies when there are no rods on the plate but its effect on the rod-induced small frequency perturbation is a higher-order small effect. Consider the following displacement field which describes the plate TS\textit{t} motion of interest:

\[ u_3 = u_3(x_3) \exp(i\omega t), \quad u_1 = u_2 = 0. \]

(1)

The nontrivial components of the strain and stress tensors are

\[ S_{33} = u_{3,3}, \quad T_{11} = T_{22} = c_{11}u_{3,3}, \quad T_{33} = c_{33}u_{3,3}, \]

(2)

(3)

where the time-harmonic factor has been dropped for convenience. The relevant equation of motion is

\[ T_{33,3} = c_{33}u_{3,3,3} = -\rho\omega^2 u_3. \]

(4)

The general solution to eq. (4) and the corresponding expression for the stress component needed in the relevant boundary and continuity conditions are

\[ u_3 = B \cos\xi(x_3 + h), \]

(5)

\[ T_{33} = -c_{33}\xi B \sin\xi(x_3 + h), \]

(6)

where \( B \) is an undetermined constant, and

\[ \xi^2 = \frac{\rho}{c_{33}}\omega^2. \]

(7)

We note that the traction-free boundary condition \( T_{33} = 0 \) at the plate bottom \( x_3 = -h \) is already satisfied by eq. (6).

2 Rod extension

For the rods, we use the one-dimensional theory for longitudinal motions [10,11]. Corresponding to the TS\textit{t} vibration of the plate, for steady-state time-harmonic motions, all rods vibrate in phase. For a typical rod (see Figure 2), let the longitudinal displacement be \( w(z) \exp(i\omega t) \), and the axial force be \( P \). We have

\[ P' = \overline{\rho}A\dot{w}, \]

(8)

\[ P = EAw', \]

(9)

where \( E \) is the Young’s modulus, \( \overline{\rho} \) is the mass density of the rod, and \( A \) is the area of the rod cross section. A prime indicates a derivative with respect to \( z \). A superimposed dot is a time derivative. It can be found in a straightforward manner that the solution to eq. (8) that satisfies the free end condition at the top of the rods, i.e., \( P(L) = EAw'(L) = 0 \), is given by

\[ w = Df(z, E, \overline{\rho}, L, \omega), \]

(10)

where \( D \) is an undetermined constant and

\[ f = \cos\alpha(z - L), \]

\[ \alpha = \omega\sqrt{\frac{\overline{\rho}}{E}}. \]

(11)

From eqs. (9) and (10) we calculate the axial force at the rod bottom as:

\[ P' = \overline{\rho}A\dot{w}, \]

\[ P = EAw', \]

\[ w = Df(z, E, \overline{\rho}, L, \omega), \]

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(11)