Abstract In the book (Adaptive Identification, Prediction and Control—Multi Level Recursive Approach), the concept of dynamical linearization of nonlinear systems has been presented. This dynamical linearization is formal only, not a real linearization. From the linearization procedure, we can find a new approach of system identification, which is on-line real-time modeling and real-time feedback control correction. The modeling and real-time feedback control have been integrated in the identification approach, with the parameter adaptation model being abandoned. The structure adaptation of control systems has been achieved, which avoids the complex modeling steps. The objective of this paper is to introduce the approach of integrated modeling and control.

Keywords Real time identification, Feedback control, Structure adaptive

1 Introduction

In order to design a model-free controller, we discussed the dynamical linearization problem of a nonlinear system in Ref. [1]. In fact, this linearization is formal. In that article, we presented that on some conditions, the following nonlinear model

\[ y(k) = f[Y_{k-1}, \cdot \cdot \cdot , Y_{k-n}, k] \] (1)

could be described in real-time by the following model combined with control law

\[ y(k) - y(k-1) = \varphi(k) [u(k-1) - u(k-2)] \] (2)

where \( y(k) \) is one-dimension system output, \( u(k) \) is multi-dimension input, \( k \) is discrete time. And

\[ Y_{k-1} = [y(k-1), \cdot \cdot \cdot , y(k-n)] \], \( n \) is a positive integer.

\[ U_{k-2}^{k-m} = \{u(k-2), \cdot \cdot \cdot , u(k-m)\} \], \( m \) is a positive integer.

\( \varphi(k) \) is a function of \( U_{k-2}^{k-m} \), \( Y_{k-1}^{k-n} \). This is called character parameter or a pseudo gradient of Eq. (1).

This dynamical linearization is only formal rather than real. But why does this linearization play an important role in the model-free control law? We found through this study that the process called dynamical linearization contains a new idea of modeling.

Generally speaking, the mathematical model of a dynamical system needs to be constituted in the process of designing a control law. The mathematical model must first be constituted in the classical method, or at least its structure must be determined first. The more accurate the model is, the better it is. In the design of a model-free controller, this restriction was removed, in which a more accurate mathematical model must be built beforehand in designing a control law.

In this case the modeling process is followed with feedback control. The initial model may be inaccurate, while convergence of the controller used must be held. The model-free controller is such a kind of controller, which models while controlling, then obtains new data, and goes back to modeling while controlling again. This process is continued and the model that we obtain from this process tends to be more precise. The property of control law is hence improved. We call this procedure the integrated approach of modeling and adaptive control.

Here, the traditional way of adaptive control law was abandoned. The traditional way is to achieve an adaptive aim by designing a control law according to a model initially built for the controlled object, and identifying the model or controlling parameter on-line. This is traditional adaptation. The disadvantage of this kind of adaptation is that it is a parameter adaptation only, because the basic structure (e.g., the order) of the model has been determined off-line at first. It is difficult to realize the structure adaptation.

The traditional way of adaptive control law appears to have more drawbacks when applied in complex systems. Because the model structure (e.g., the order) has been determined off-line, it is difficult to satisfy the requirements of some differences in system structure.
2 Universal model and character parameter

The following universal model has been presented in Refs. [1-2],

\[ y(k) - y(k-1) = \varphi(k-1)'[u(k-1) - u(k-2)] \]  

(3)

Without losing generality, suppose that the time-delay of controlled system is 1, \( y(k) \) is a one-dimension system output, \( u(k-1) \) is a p-dimension input vector, \( \varphi(k-1) \) is character parameter, which can be on-line estimated using some identification method, and \( k \) is discrete time. It will show that: in the integrated approach of real-time identification, real feedback correction \( \varphi(k) \), has clear mathematics and engineering significance.

Suppose a dynamical system can be denoted in the following form:

\[ y(k) = f[Y_{k-1}^{k-n}, u(k-1), U_{k-2}^{k-n}, k] \]  

(4)

where \( Y_{k-1}^{k-n}, U_{k-2}^{k-n} \) is the same with the above meaning. Assume that \( f[Y_{k-1}^{k-n}, u(k-1), U_{k-2}^{k-n}, k] \) has a continuous gradient with respect to \( u(k-1) \). When the system \( S \) is at a steady state, assume it satisfies the condition that if \( u(k-1) = u(k-2) \), then \( y(k) = y(k-1) \) (in random case, \( E[y(k)] = E[y(k-1)] \)).

Hence,

\[ y(k) - y(k-1) = f[Y_{k-1}^{k-n}, u(k-1), U_{k-2}^{k-n}, k] - f[Y_{k-1}^{k-n}, u(k-2), U_{k-2}^{k-n}, k] = f[Y_{k-1}^{k-n}, u(k-1), U_{k-2}^{k-n}, k] - f[Y_{k-1}^{k-n}, u(k-2), U_{k-2}^{k-n}, k] \]

Let

\[ f[Y_{k-1}^{k-n}, u(k-2), U_{k-2}^{k-n}, k] - f[Y_{k-1}^{k-n}, U_{k-2}^{k-n}, k] = \zeta(k) \]

Using the mean value theorem in the calculus, we obtain:

\[ f[Y_{k-1}^{k-n}, u(k-1), U_{k-2}^{k-n}, k] - f[Y_{k-1}^{k-n}, u(k-2), U_{k-2}^{k-n}, k] \]

\[ = \nabla_{u(k-2)} f[Y_{k-1}^{k-n}, u(k-2), U_{k-2}^{k-n}, k]'[u(k-1) - u(k-2)] \]

where \( \nabla_{u(k-2)} f[Y_{k-1}^{k-n}, u(k-2), k] = \nabla_{u(k-2)} f[Y_{k-1}^{k-n}, u(k-2), U_{k-2}^{k-n}, k] \)

(5)

Therefore we have:

\[ \nabla_{u(k-2)} f[Y_{k-1}^{k-n}, u(k-2), U_{k-2}^{k-n}, k] \]

(6)

where \( \nabla_{u(k-2)} f[Y_{k-1}^{k-n}, u(k-2), U_{k-2}^{k-n}, k] = \nabla_{u(k-2)} f[Y_{k-1}^{k-n}, U_{k-2}^{k-n}, k] \)

**Definition 1** If from conditions \( Y_{k-1}^{k-n}, U_{k-2}^{k-n}, u(k-1), u(k-2) \), it can be obtained that:

\[ f[Y_{k-1}^{k-n}, u(k-1), U_{k-2}^{k-n}, k] = f[Y_{k-1}^{k-n}, u(k-2), U_{k-2}^{k-n}, k] \]

then system (4) is an auto time-invariant system.

Obviously, if system (4) is auto time-invariant and the function \( f[Y_{k-1}^{k-n}, u(k-1), U_{k-2}^{k-n}, k] \) is continuous with respect to \( Y_{k-1}^{k-n}, u(k-1), U_{k-2}^{k-n}, k \), we certainly have

\[ \lim_{u(k-1) \to u(k-2), t 

Especially, we have

\[ \lim_{u(k-1) \to u(k-2), t 

Frequently, if we let \( \varphi(k) = \nabla_{u(k-2)} f[Y_{k-1}^{k-n}, u(k-2), U_{k-2}^{k-n}, k] \), then Eq. (6) can be written as:

\[ y(k) - y(k-1) = \varphi(k)'[u(k-1) - u(k-2)] + \zeta(k) \]

(7)

If \( \|u(k-1) - u(k-2)\| = 0 \), let

\[ \varphi(k) = \psi(k) + \frac{u(k-1) - u(k-2)}{\|u(k-1) - u(k-2)\|} \]

then Eq. (6) can be rewritten as:

\[ y(k) - y(k-1) = \varphi(k-1)'[u(k-1) - u(k-2)] \]

(8)

**Note:** When the system is in a steady state, when \( \|u(k-1) - u(k-2)\| = 0 \), we have \( y(k) = y(k-1) \), in this case, Eq. (7) can be considered naturally valid.

Eq. (7) is the universal model, which has been presented. \( \varphi(k-1) \) is the character parameter vector.

3 Integration of real time modeling and feedback controlling

It is clear that the necessary condition for the universal model:

\[ y(k) - y(k-1) = \varphi(k-1)'[u(k-1) - u(k-2)] \]

(9)

to be used in practice is that the estimate value \( \hat{\varphi}(k-1) \) of \( \varphi(k-1) \) can be obtained in real-time with enough accuracy. As for estimating \( \varphi(k-1) \), there are several ways. For example:

1. **Recursive least square**

Let

\[ z(k) = y(k) - y(k-1) \]

\[ \hat{\varphi}(k) = u(k-1) - u(k-2) \]

the Eq. (8) becomes

\[ z(k) = \hat{\varphi}(k)'\hat{\varphi}(k-1) \]

(10)

when \( y(k), u(k-1) \) are observed in real time, \( z(k) \) and