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Entanglement reciprocation between two charge qubits and two-cavity field

Abstract We propose a simple scheme to generate two-mode entangled coherent state in two separated cavities and realize the entanglement reciprocation between the superconducting charge qubits and continuous-variable system. By measuring the state of charge qubits, we find that the entanglement of two charge qubits, which are initially prepared in the maximally entangled state, can be transferred to the two-cavity field, and at this time the two-cavity field is in the entangled coherent state. We also find that the entanglement can be retrieved back to the two charge qubits after measuring the state of the two-cavity field.

Keywords entanglement transfer, entangled coherent state, superconducting quantum interference devices

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1 Introduction

Quantum entanglement and entangled states are useful in quantum information processing such as quantum cryptography [1, 2], superdense coding [3], and tele-coloning [4]. Especially, entangled states of the electromagnetic field are of particular interest. Such states can be used, e.g., for quantum key distribution [2] and teleportation [5]. There are different types of entanglement for light fields. For instance, in the teleportation experiments of the Innsbruck group [6], it is the polarization directions of single photon that are entangled. In the Caltech teleportation experiment [7], two electromagnetic field modes are entangled with respect to photon numbers and the state used for teleportation is two-mode squeezed state.

Besides, there is another type of entangled states of two modes of the electromagnetic field. They are called entangled coherent states (ECS) [8, 9], and using these states people have proposed some schemes to teleport quantum states [10–12]. The ECS can be expressed as

$$|n^\pm\rangle = N_{\pm}(|\alpha, -\alpha\rangle \pm |-\alpha, \alpha\rangle)$$

where the normalization constants $N_{\pm} = (2 \pm 2e^{-4|\alpha|^2})^{-1/2}$, $|\alpha\rangle$ is the coherent state and $\alpha$ is a complex parameter. It has been pointed out that the concurrence $C$ of the ECS $|n^+\rangle$ increases with the increasing of $\alpha$, and as $|\alpha| \to \infty$, $C \to 1$. While the amount of entanglement of the state $|n^-\rangle$ is independent of the parameter involved, and the concurrence of this state is always equal to 1 [8–10]. Recently, there has been increasing interest in the generation of entangled coherent states [8, 13–19]. Kuang and Zhou have proposed a scheme to generate atom-photon ECS in atomic Bose-Einstein condensation via electromagnetically induced transparency [13]. The generation of high-dimensional photon ECS in a double electromagnetically induced transparency system was proposed by Guo and Kuang [14]. Gerry [17, 18] was the first to propose an elegant scheme to realize GHZ-type ECS using three separated cavities. In Ref. [19], Solano et al. proposed a scheme to generate two-mode ECS in a cavity.

In recent years, entanglement transfer from the continuous-variable (CV) system to the qubits has been
widely studied [20–23], and the inverse problem of entanglement transfer from the qubits to the CV system has also been discussed [24, 25]. Lee et al. [24] have investigated the entanglement reciprocation between the qubits and the CV system. They considered that two atoms, which are initially prepared in a maximally entangled state, enter into two spatially separated cavities, which are respectively prepared in a coherent state. It was shown that when the two atoms leave the cavities, their entanglement is transferred to the post-selected cavity fields. The generated field entanglement can be then transferred back to qubits, i.e., to another couple of atoms flying through the cavities. Zhou et al. [25] have also studied the entanglement transfer between atomic qubits and the CV system. They considered the model of that two identical atoms interacting with two spatially separated cavities, while the two atoms are driven by two classical fields respectively. It has been found that entanglement can be reciprocated between atomic qubits and entangled coherent state via postselection measurement.

Recently, condensed matter architectures based on Josephson junction qubits have appeared to be promising candidates for quantum information processing. Superconducting qubit has been successfully incorporated into a superconducting resonant cavity in order to perform analogous experiments in the strong coupling regime, forming a new field known as circuit QED [26]. In Ref. [27], Liu et al. have investigated the interaction between a single-mode microwave cavity field and a superconducting quantum interference device (SQUID). After some calculation they found that by measuring the charge state $|e\rangle$ or $|g\rangle$, the superpositions of macroscopic states of the cavity field can be produced. Following Ref. [27], we consider that two identical SQUID-type qubits 1 and 2 are in two spatially separated but identical cavities $A$ and $B$ respectively. First, the two charge qubits are initially prepared in the singlet state $\frac{1}{\sqrt{2}}(|eg\rangle - |ge\rangle)$, and we investigate the entanglement transfer from the two charge qubits to the two-cavity field. After some calculation we find that the two-cavity field, which is initially in vacuum state $|00\rangle$, can be prepared into ECS by entanglement transfer. Then we can fully retrieve the entanglement back to the two charge qubits from the ECS of the two-cavity field.

2 Model

Following Ref. [27], we consider that two identical SQUID-type qubits 1 and 2 are in two spatially separated but identical cavities $A$ and $B$ respectively [see Fig. 1(a)], and this setup has been used to investigate the entanglement transfer from the nonclassical state of the cavity field to a pair of superconducting charge qubits [21]. A SQUID-type qubit superconducting box [see Fig. 1(b)] with $n$ excess Cooper-pair charges connects to a superconducting loop via two identical Josephson junctions with capacitors $C_j$ and coupling energies $E_j$. A controllable gate voltage $V_g$ is coupled to the box via the gate capacitor $C_g$, with dimensionless gate charge $n_g = C_g V_g / (2e)$. When $n_g = 1/2$, only two charge states ($n = 0$ and $n = 1$) play a leading role, so the superconducting box can be considered as a two-level system or qubit. This superconducting two-level system can be represented by a spin-1/2 notation such that the charge states $n = 0$ and $n = 1$ correspond to eigenstates $|\uparrow\rangle$ and $|\downarrow\rangle$ of the spin operator $\sigma_z$, respectively. The ground and excited states for the qubit are respectively denoted by $|g\rangle = |\uparrow\rangle = (|+\rangle + |-\rangle)/2$ and $|e\rangle = |\downarrow\rangle = (|+\rangle - |-\rangle)/2$ where $|+\rangle (-\rangle)$ is eigenstate of the Pauli operator $\sigma_x$ with the eigenvalue $1 (-1)$. In this model, the Hamiltonian of the whole system can be written as

$$H = H_{1,A} + H_{2,B}$$

where $H_{1,A}$ and $H_{2,B}$ are the effective Hamiltonians for the interaction between each qubit and its cavity. The effective Hamiltonian for the interaction between qubit 1 and cavity $A$ has the form [27]

![Fig. 1](image-url)