Andreev reflection and tunneling spectrum on metal–superconductor–metal junctions

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The tunneling spectrum of an electron and a hole in metal–superconductor–metal junctions is computed using the Blonder–Tinkham–Klapwijk method. The incident and the outgoing currents finally balance each other by an interface charge inside the superconductor and metal junction. The present computation shows a more abundant structure compared to that on a metal–superconductor junction, such as the resonance at bias voltages above the energy gap of the superconductor. The density of the interface charge shows a quantum-like oscillation.

Keywords Andreev reflection, superconductivity

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1 Introduction

The research on the Andreev reflection in metal–superconductor junctions has a long history and still attracts great interests of scientists recently [1–3]. The tunneling spectrum of electrons on a metal–superconductor (SC) junction is sensitive to the energy gap of the superconductor, thus becoming an important technique to measure the gaps and gap properties of superconductors. When an electron tunnels into a superconductor to form a Cooper pair there appears a hole reflection to conserve the charges, which is called the Andreev reflection. A commonly used theoretical method to investigate the Andreev reflection is the Blonder–Tinkham–Klapwijk (BTK) approach [1], which takes the interface in a metal–superconductor junction as a $\delta(x)$ potential barrier. This theory has been widely and successfully applied to systems like metal–superconductor junctions [4], ferromagnet–SC junctions [5], etc. Wei, Dong and Xing et al. studied the tunneling spectrum in metal–SC–metal (MSM) junctions using the BTK approach [6, 7]. They found that the incident and outgoing currents in both sides do not balance each other [6]. Thus they claimed at an earlier time that the BTK approach are not suitable for the Andreev reflection in MSM junctions and treated them in the Landauer–Büttiker formalism [8]. But later they balanced the current by adjusting the chemical potential of the superconductor inside the MSM junction.

The quasi-particle current and superconducting current are continuous inside the superconductor everywhere, hence, the charge density inside the superconductor does not change according to the continuity equation of charges. Therefore, the chemical potential thus the Fermi surface of the SC inside MSM junctions may not change. On the outgoing interface, however, the current does not balance on the two sides. Therefore, there must be a charge accumulation on this interface. This interface charge density changes the potential of the metal in the outgoing side and finally reaches a current balance. This is the main idea of this paper. Through detailed computation we demonstrate that this mechanism is reasonable. The charge accumulation should be a real effect in MSM junctions and can be measured in experiments. This dynamic process can be simulated by solving the time-dependent Bogoliubov–de Gennes (BdG) equations. In the present paper a simpler model is set up to describe the final balanced state of the MSM junctions.

2 Formalism

The MSM junctions are shown in Fig. 1, where two thin insulating layers exist inside the two junctions which become two potential barriers and are treated as a $\delta(x)$ potential in the BTK approach. An electron and a hole are
incident into the junctions under a bias voltage on both sides. Due to the two barriers the electron and the hole are partially reflected and partially transmitted into the SC. In the SC a quasi-particle can only propagate above the energy gap $\Delta$. When the bias voltage is smaller than $\Delta$ only Cooper pairs can propagate in the SC. In this case there will be a hole reflected on both sides, which are called Andreev reflection in literatures.

\[
\frac{i\hbar}{\partial t} \begin{pmatrix} u(r, t) \\ v(r, t) \end{pmatrix} = \begin{pmatrix} H_0 & \Delta \\ \Delta^* & -H_0 \end{pmatrix} \begin{pmatrix} u(r, t) \\ v(r, t) \end{pmatrix}
\]

(1)

where $H_0 = -\frac{\hbar^2}{2m} \nabla^2 + V(r) - \mu$.

The tunneling process of an electron and a hole in the MSM junctions is described by the following Bogoliubov–de Gennes (BdG) equation \[9\]

\[
\frac{\partial \rho_Q(r)}{\partial t} + \nabla \cdot (J_Q(r) + J_S(r)) = 0
\]

(3)

where $\rho_Q(r)$ is the charge density and $J_Q(r) = \frac{4e}{\hbar} Im(u^* \nabla u + v^* \nabla v)$ the current density of quasi-particles. The supercurrent density is defined by

\[
\nabla \cdot J_S(r) = -\frac{4e}{\hbar} Im(\Delta u^*(r)v(r))
\]

(4)

It can be proved that quasi-particle current is continuous along the whole MSM junctions, but the supercurrent is obviously discontinuous since no supercurrent exists in a metal. This charges the interface like the charging process on a capacitor with a bias voltage. The charging process can be seen by integrating the continuity equation over a slab volume across the interface, resulting in

\[
\frac{\partial \sigma(t)}{\partial t} = J_S(x = L)
\]

(5)

where $\sigma(t)$ is the surface charge density on the interface and $J_S(x = L)$ is the supercurrent at the interface. The interface charge distribution changes the potential of electrons and holes in metals II, and therefore the outgoing current is changed and finally balances with the incident current.

As seen from Fig. 2, when the bias voltage satisfies $eV > \Delta$ an incident particle in metals tunnels into the SC through an interface and propagate as a quasi-particle. It is then partially reflected by the following the following interface and partially tunnels as the following metal. The wave functions of the electron and hole inside the metals and the SC are written as

\[
\psi_I = \begin{pmatrix} e^{ik^+ x} + be^{-ik^+ x} \\ a e^{ik^- x} \end{pmatrix}
\]

(6)

\[
\psi_{SC} = \begin{pmatrix} e^{iq^+ x} \begin{pmatrix} u^+ \\ v^+ \end{pmatrix} + de^{-iq^- x} \begin{pmatrix} u^- \\ v^- \end{pmatrix} \\ +ee^{-iq^- x} \begin{pmatrix} u^+ \\ v^+ \end{pmatrix} + f e^{iq^+ x} \begin{pmatrix} u^- \\ v^- \end{pmatrix} \end{pmatrix}
\]

(7)

\[
\psi_{II} = \begin{pmatrix} g e^{ik^+ x} \\ e^{ik^- x} + he^{-ik^- x} \end{pmatrix}
\]

(8)

where $(u^+)$ and $(u^-)$ are the quasi-electron and quasi-hole wave functions, respectively. They are given by

\[
\begin{align*}
 u_+ &= \sqrt{\frac{1}{2} \pm \frac{\varepsilon_{q\pm} - \mu}{2E}}, & v_+ &= \frac{|\Delta|}{\Delta} \sqrt{\frac{1}{2} \pm \frac{\varepsilon_{q\pm} - \mu}{2E}} \\
 E &= \sqrt{(\varepsilon_{q\pm} - \mu)^2 + |\Delta|^2}, & \varepsilon_{q\pm} &= \frac{\hbar^2 k_{q\pm}^2}{2m}
\end{align*}
\]

(9)

(10)

The wave vectors are determined by

\[
\begin{align*}
 E &= eV = \frac{\hbar^2 k_x^2}{2m} - \mu = \mu - \frac{\hbar^2 k_y^2}{2m} \\
 k_{\pm} / k_F &= \sqrt{1 \pm \frac{E^2 - \Delta^2}{\mu}}
\end{align*}
\]

(11)

(12)

(13)