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\( \sqrt{2} \) Rule for Controlling the Tree Pattern in Forest Cut

Abstract A forest’s productivity can be optimized by the application of rules derived from monopolized circles. A monopolized circle is defined as a circle whose center is a tree and whose radius is half of the distance between the tree itself and its nearest neighbor. Three characteristics of monopolized circle are proved. (1) Monopolized circles do not overlay each other, the nearest relationship being tangent. (2) “Full uniform pattern” means that the grid of trees \((a \times b = N)\) covers the whole plot, so that the distance between each tree in a row is the same as the row spacing. The total monopolized circle area with a full uniform pattern is independent on the number of trees and \(\pi/4\) times the plot area. (3) If a tree is removed, the area of some trees’ monopolized circle will increase without decreasing the monopolized circles of the other trees. According to the above three characteristics, “uniform index” is defined as the total area of monopolized circles in a plot divided by the total area of the monopolized circles, arranged in a uniform pattern in the same shaped plot. According to the definition of monopolized circle, the distribution of uniform index \((L) = \frac{\pi a^2}{4n}\) for a random pattern and \(E(L) = \frac{1}{2}\); the variance of \(L\) is \(D(L) = \frac{1}{16\pi}\). It is evident that \(E(L)\) is independent on \(N\) and the plot area; hence, \(L\) is a relative index. \(L\) can be used to compare the uniformity among plots with different areas and the numbers of trees. In a random pattern, where \(L\) is equivalent to the tree density of the plot in which the number of trees is 1 and the area is \(\pi\), the influence of tree number and plot area to \(L\) is eliminated. When \(n \rightarrow \infty\), \(D(L) \rightarrow 0\) and \(L \rightarrow \frac{1}{\pi} = 0.318\); it indicates that the greater the number of tree is in the plots, the smaller the difference between the uniform indices will be. There are three types of patterns for describing tree distribution (aggregated, random, and uniform patterns). Since the distribution of \(L\) in the random pattern is accurately derived, \(L\) can be used to test the pattern types. The research on Moarshan showed that the whole plot has an aggregated pattern; the first, third, and sixth parts have an aggregated pattern; and the second, fourth, and fifth parts have a random pattern. None of the uniform indices is more than 0.318 \((1/\pi)\), which indicates that uniform patterns are rare in natural forests. The rules of uniform index can be applied to forest thinning. If you want to increase the value of uniform index, you must increase the total area of monopolized circles, which can be done by removing select trees. “Increasing area trees” are the removed trees and can increase the value of the uniform index. A tree is an increasing area tree if the distance between the tree and its second nearest neighbor is \(\sqrt{2}\) times longer than that between the tree itself and its first nearest neighbor, which is called the \(\sqrt{2}\) rule. It was very interesting to find that when six plots were randomly separated from the original plot, the proportion of increasing area trees in each plot was always about 0.5 without exception. In random pattern, the expected proportion of increasing area trees is about 0.35–0.44, which is different from the sampling value of 0.5. The reason is very difficult to explain, and further study is needed. Two criteria can be used to identify which trees should be removed to increase the uniform index during forest thinning. Those trees should be (1) trees whose monopolized circle areas are on the small side and (2) increasing area trees, which are found via the \(\sqrt{2}\) rule.

Keywords forest ecology, monopolized circle, pattern, Poisson distribution, pattern control, uniform index


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1 Introduction


The Moristita index and the variance mean are universally applied in pattern research [13–15]. Population pattern is related to the state of mobile sand and plant habit in Maowusu tibba [16]. Under bad ecological conditions, the plant has a discontinuously aggregated distribution [17]. Zhang [18] used the same method to analyze point pattern.

There are two sampling methods in population pattern research. One is applied to a small-shaped, large-sized population and the other to a large-shaped, small-sized population. Distance sampling is suitable to the latter. Based on this, the author brings forward the notion of monopolized circle and the method in measuring and testing pattern homogeneity with uniform index.

The study area (127°28′–127°43′E, 45°14′–45°29′N) is located in Moarshan, on the west side of the Zhangguangcai range, Changbai mountain system, China. Mean elevation of the area is 300 m, and secondary forest is dominant [19]. There are many types of dense canal systems that influence forest macropattern [20]. The study on birch population spatial pattern indicates that a youngling forest has an aggregated distribution, whereas an adult one has a random distribution [21].

2 Correlative lemma of monopolized circle and definition of uniform index

**Nth monopolized circle** The distance between a plant body at random and its Nth nearest neighbor is defined as $S$; the circle whose radius is $S/2$ and whose center is this plant is called the Nth monopolized circle. Obviously, the first monopolized circle is a monopolized circle.

**Lemma 1** In a plot without plant invasion, monopolized circles of all individuals do not overlap.

**Proof** Reduction to absurdity is used. Suppose that there are two overlapping monopolized circles and they are not tangent. Let A and B be the centers of the two circles, $R_A$ and $R_B$ be the radii, respectively, and $R_A \leq R_B$. Then, $R_A + R_B > AB$. Following the definition of monopolized circle, the distance between A and its nearest body is $2R_A$, whereas $2R_A + R_B > AB$. It means that the distance between A and B is less than the nearest body distance of A. It is in contradiction to the above hypothesis.

**Lemma 2** Let uniformly distributed plants in a rectangle plot be $a \times b = n$, and the distance between every plant and its nearest neighbor be $s$. This is called complete uniform pattern. When the stem number of a sample area is $n$, $4n, ... 4^n$, the total area of a monopolized circle is invariable.

**Proof** From the above hypothesis, a monopolized circle is a circle whose center is a random plant and whose radius is $S/2$. There is a tangent square outside the monopolized circle whose side is $s$. Obviously, all tangent squares cover the whole plot. Thus, the total sample area is $ns^2$, and the total area of the monopolized circle is $n \frac{\pi s^2}{4}$. If we divide every tangent square into four and put a plant in the center of every small square, the total sample area is $4n \left( \frac{s}{2} \right)^2 = ns^2$.

Then, the total area of the monopolized circle is $4n \pi \left( \frac{s}{2} \right)^2 = n \frac{\pi s^2}{4}$, where the stem number is $4n$. Therefore, for complete uniform pattern, when the stem number is $n, 4n, ... 4^n$, the total area of the monopolized circle is invariable, and it is $\frac{\pi}{4}$ times the total sample area.

### 2.1 Definition of uniform index

On the basis of Lemma 1, monopolized circles do not overlap. The summation of its area is decided by pattern.

On the basis of Lemma 2, for uniform distribution, the total area of a monopolized circle is independent on the stem number, and it is $\frac{\pi}{4}$ times the sample area.

From the two lemmas, we can deduce the definition of uniform index.

In a plot, the ratio of the total area of every plant’s monopolized circle to $\frac{\pi}{4}$ times the sample area is called the uniform index.

Suppose a is the total monopolized circle area of a plant, $A$ is the sample area, and $L = \frac{4a}{\pi A}$ is the uniform index. Then, $a = \frac{\pi AL}{4}$. 

**Nth near body** The nearest tree $A_1$ apart from tree A is called the first nearest body of A; the second nearest tree $A_2$ is called the second nearest body, the $N$th nearest tree apart from A is called the $N$th nearest body of A.

**Nearest body** The nearest body $A_1$ apart from body A is called the nearest body; that is, the first nearest body and nearest body are equivalent.

**Monopolized circle** The distance between a plant body at random and its nearest neighbor is defined as $S$; then, the circle whose radius is $S/2$ and whose center is this plant is called a monopolized circle.