Hot topics in cold gases*
A mathematical physics perspective

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Abstract. We present an overview of mathematical results on the low temperature properties of dilute quantum gases, which have been obtained in the past few years. The presentation includes a discussion of Bose–Einstein condensation, the excitation spectrum for trapped gases and its relation to superfluidity, as well as the appearance of quantized vortices in rotating systems. All these properties are intensely being studied in current experiments on cold atomic gases. We will give a description of the mathematics involved in understanding these phenomena, starting from the underlying many-body Schrödinger equation.

Keywords and phrases: quantum statistical mechanics, Bose–Einstein condensation, dilute Bose gas, superfluidity, excitation spectrum

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1. Introduction

Bose–Einstein condensation (BEC) in cold atomic gases was first achieved experimentally in 1995 [1,7]. After initial failed attempts with spin-polarized atomic hydrogen, the first successful demonstrations of this phenomenon used gases of rubidium and sodium atoms, respectively. Since then there has been a surge of activity in this field, with ingenious experiments putting forth more and more astonishing results about the behavior of matter at very cold temperatures. BEC has now been achieved by more than a dozen different research groups working with gases of different types of atoms. Literally thousands of scientific articles, concerning both theory and experiment, have been published in recent years.

The theoretical investigation of BEC goes back much further, and even pre-dates the modern formulation of quantum mechanics. It was investigated in two papers by Einstein [11] in 1924 and 1925, respectively, following up on a work by Bose [3] on the derivation of Planck’s radiation law. Einstein’s result, in its modern formulation, can be found in any textbook on quantum statistical mechanics, and was concerned with ideal, i.e., non-interacting gases.

The understanding of BEC in the presence of interparticle interactions poses a formidable challenge to mathematical physics. Some progress was made in the last decade, and the purpose of these lecture notes is to explain part of what was achieved and how it is related to the actual experiments on cold gases. The