Vacuum Effects over the Closing of Enterocutaneous Fistulae: A Mathematical Modeling Approach

D.I. Cattoni\textsuperscript{a,b}, O. Chara\textsuperscript{c,d,*}

\textsuperscript{a} Cátedra de Física, Departamento de Fisicomatemática, Facultad de Farmacia y Bioquímica, UBA, Buenos Aires, Argentina
\textsuperscript{b} Instituto de Química y Fisicoquímica Biológica, Facultad de Farmacia y Bioquímica, UBA, Buenos Aires, Argentina
\textsuperscript{c} Instituto de Física de Líquidos y Sistemas Biológicos (IFLYSIB), La Plata, Argentina
\textsuperscript{d} Departamento de Fisiología y Biofísica, Facultad de Medicina, UBA, Buenos Aires, Argentina

Received: 26 October 2006 / Accepted: 14 June 2007 / Published online: 15 August 2007
© Society for Mathematical Biology 2007

Abstract Enterocutaneous fistulae are pathological communications between the intestinal lumen and the abdominal skin. Under surgery the mortality of this pathology is very high, therefore a vacuum applying system has been carried previously on attempting to close these fistulae. The objective of this article is the understanding of how these treatments might work through deterministic mathematical modelling. Four models are here proposed based on several assumptions involving: the conservation of the flow in the fistula, a low enough Reynolds number justifying a laminar flow, the use of Poiseuille law to model the movement of the fistulous liquid, as well as phenomenological equations including the fistula tissue and intermediate chamber compressibility. Interestingly, the four models show fistulae closing behaviour during experimental time ($t < 60$ sec). To compare the models, both, simulations and pressure measurements, carried out on the vacuum connected to the patients, are performed. Time course of pressure are then simulated (from each model) and fitted to the experimental data. The model which best describes actual measurements shows exponential pumping flux kinetics. Applying this model, numerical relationship between the fistula compressibility and closure time is presented. The models here developed would contribute to clarify the treatment mechanism and, eventually, improve the fistulae treatment.

Keywords Fistula · Mathematical model · Low pressure · Elastic modulus · Simulation · Compressibility

*Corresponding author.
E-mail addresses: ochara@iflysib.unlp.edu.ar; ochara@fmed.uba.ar (O. Chara).
1. Introduction

Enterocutaneous fistulae are pathological communications between the lumen of the intestine and the external abdominal skin. The therapy employed to treat this pathology is usually surgery. However, even with this medical intervention, the mortality is very high (Altomare et al., 1990; Campos et al., 1999; Levy et al., 1989; Lynch et al., 2004).

In order to complement the surgery several authors have studied the effects of sub-atmospheric pressure on biological tissues (Alvarez et al., 2001; Argenta and Morykwas, 1997; Ballard and Baxter, 2001; Miller et al., 2002; Webb, 2002).

Previously a treatment was developed to close enterocutaneous fistulae based in a vacuum and compaction system (Fernández et al., 1992) and vacuum-assisted closure (Erdmann et al., 2001). Briefly, the abdominal surface containing the fistulae atmospheric extreme is covered by polymeric foam and this by a nylon film.

This compartment will be referred in this article as the compaction chamber. This chamber is connected by a catheter to a vacuum pump. Shortly after generating the vacuum (12 ± 5 seconds, \( n = 15 \) patients) a transitory closing of the fistulae is achieved (Fig. 1). Eventually some fistulae can be closed permanently. This method is carried out nowadays at the Tornu Hospital in Buenos Aires, Argentina, with optimal results (Wainstein et al., 2005).

In this article, deterministic mathematical models of enterocutaneous fistulae exposure to vacuum are developed for the first time.

The models are based in the ten assumptions detailed below (see Assumptions). The intestines, the fistula, the compaction chamber and the connection with the pump are modelled as a succession of compartments all connected in series (Fig. 2). From this first order non linear differential equation are obtained. Finally, simulations are carried on to solve the system numerically by the Euler method. In order to do this, an algorithm is implemented in a FORTRAN 90 code to simulate the behaviour of the fistulae under vacuum application.

The main objective of this work is to comprehend, through mathematical modelling, why low pressure could be appropriate to treat enterocutaneous fistulae.

2. Assumptions

The general models are schematized in Fig. 2. In this figure, it can be observed a graph (from the point of view of the graph theory). This graph show three vertices (1, 0 and \(-1\),