Modeling Optimal Intervention Strategies for Cholera

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1. Introduction

1.1. Background

Cholera is a diarrhoeal illness caused by infection of the intestine with the bacterium Vibrio cholerae. Food and water supplies contaminated with V. cholerae are the primary source of disease spread. While cholera has been a recognized disease for about 200 years, the control of deadly outbreaks remains a challenge. Highlighted by the 2008 outbreak in

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All authors worked on the model and optimal control formulation as well as the interpretation of the results. Miller Neilan contributed the optimal control characterization and numerical results. Schaefer contributed the parameter sensitivity analysis.
Zimbabwe, a new focus on controlling cholera has emerged due to a rise in cholera incidence. The recent outbreaks have caused enormous loss of life and financial devastation to families and healthcare systems (World Health Organization, 2008).

A complex system of interactions occurs between the human host, pathogen, and environment. Merrell and Butler (2002) reported a study showing freshly shed V. cholerae from human intestines outcompetes other V. cholerae by as much as 700-fold for at least the first 5 hours in the environment. This prompted Hartley et al. (2006) to modify an earlier model by Codeço (2001) to explicitly account for the concentration of hyperinfectious bacteria within drinking water. Associating higher levels of hyperinfectious concentrations with increased human-to-human transmission, the authors illustrate the importance of this additional state in modeling the explosive epidemics often associated with cholera (Hartley et al., 2006).

More recently, King et al. (2008) proposed a two-path cholera model including a class for severe infections as well as a class for mild or inapparent infections. Log likelihood estimates using 1900’s mortality data in 26 Bengal districts were generated by King et al. and indicate that the districts of Bogra and Calcutta are well described by the two-path model, yet differ greatly in several parameter estimates, most notably the estimated proportion of infections resulting in severe symptoms (approximately 0.24 and 0.02, respectively). In addition to emphasizing the need for two infectious classes, the estimates show that natural immunity to cholera may wane within a year, as opposed to the previously suspected period of 3–10 years.

1.2. Initial model and objectives

Our objective is to formulate a model for cholera that includes relevant biological detail, accounts for multiple intervention strategies, and allows optimal control methods to be used. To begin, we integrate the aforementioned essential components into one SIR-type (Susceptible-Infectious-Recovered) model to accommodate the diverse dynamics of a cholera outbreak determined by population-specific parameters such as the ratio of mild-to-severe infections and the rates of bacteria ingestion. Figure 1 diagrams our initial model which is interpreted as a system of six ordinary differential equations. The figure shows two classes of bacterial concentrations, one that is hyperinfectious (class $B_H$) and one that is less-infectious (class $B_L$). Bacteria transition from the hyperinfectious state to the less-infectious state at rate $\chi$, and the less-infectious bacteria lose viability at rate $\delta$. Susceptible human individuals (class $S$) become infected with cholera through ingestion of bacteria contaminated water at rates $\beta_H$ and $\beta_L$, with the bacteria concentrations measured with respect to their infectious doses, $\kappa_H$ and $\kappa_L$. A proportion $p$ of the infected individuals experience mild or inapparent symptoms and are classified as asymptomatic infections (class $I_A$). The remaining infected individuals experience severe symptoms and are classified as symptomatic infections (class $I_S$). The asymptomatic class will assume smaller rates of cholera-related death ($e_1$) and bacterial shedding ($\eta_1$) and a larger recovery rate ($\gamma_1$) than that of the symptomatic class (rates $e_2$, $\eta_2$, and $\gamma_2$, respectively). All immunity to cholera is assumed to wane at rate $\omega$. For our short-term analysis of disease spread, natural birth and death terms are excluded.

To determine the role of the model parameters in predicting outbreak dynamics, we first conduct a sensitivity analysis using the initial model described by Fig. 1. Cholera outbreak dynamics are then simulated according to our model for two diverse endemic