Hilbert-Huang transform and wavelet analysis of time history signal*

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Abstract
The brief theories of wavelet analysis and Hilbert-Huang transform (HHT) are introduced firstly in the present paper. Then several signal data were analyzed by using wavelet and HHT methods, respectively. The comparison shows that HHT is not only an effective method for analyzing non-stationary data, but also is a useful tool for examining detailed characters of time history signal.

Key words: Hilbert-Huang transform; wavelet analysis; mother wavelet; intrinsic mode functions; spectral analysis
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Introduction

Wavelet analysis is a temporal-frequency analysis method. Traditional signal analysis is based on Fourier transform, while Fourier transform cannot express the temporal-frequency characteristic of signal which is basic and key to non-stationary signal, because it uses a global transform either completely in time domain or completely in frequency domain. Aiming to analyze and process non-stationary signal, researchers extend and reform Fourier analysis, put forward series of new theories, wavelet analysis method is among them.

Wavelet analysis with high resolution is a temporal-scale or temporal-frequency analysis method, which means discrete transform is made in time domain and spectra analysis is made in temporal-frequency domain, and it can obtain the local characteristics of signal both in time domain and in frequency domain. Wavelet analysis is known as microscope and telescope, for it has higher frequency resolution and lower time resolution in low-frequency part and higher time resolution and lower frequency resolution in high-frequency part, which is suit for detecting instant abnormal phenomena smuggling in normal signal, but it uses different wavelet function in wavelet analysis, which means the multiplicity of wavelet function. The choose of wavelet base (or mother wavelet) is an important problem for wavelet analysis applied in engineering, because using different mother wavelets to analyze same problem will produce different results. At present, mainly by comparing the error between results got by wavelet analysis and theoretical results to judge mother wavelet and to select mother wavelet (HU, et al., 1999; ZHU, 1996a, b). So wavelet analysis results are limited by mother wavelet, and wavelet components and wavelet spectra are meaningful only to the selected mother wavelet. Although these problems exist in wavelet analysis, it is still an effective method to analyze non-stationary data.

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Hilbert-Huang transform (HHT, Huang, et al, 1998) is also an effective tool suitable for analyzing non-stationary time history signal. HHT can effectively decompose seismic wave into intrinsic mode functions with different frequency from time history curve. Hilbert spectrum in HHT is a three-dimensional spectrum (HU, 1997) including time, frequency and amplitude. It can analyze energy distribution with different time and instant frequency of seismic wave. We have compared HHT with Fourier transform, and discussed the end swing problem in Huang’s transform and its resolving method (LUO, SHI, 2003). This paper will discuss applicability and capacity of HHT in analyzing non-stationary data referring to wavelet analysis.

1 Wavelet analysis

1.1 Wavelet function

For processing adjustable window analysis in wavelet analysis, the selected base function must have retractable characteristics. So base function in wavelet transform can be selected as following equation (ZHU, 1996a, b):

$$W_{s,\tau}(t) = \frac{1}{\sqrt{s}} W\left(\frac{t-\tau}{s}\right) \quad s > 0$$

(1)

where $s$, $\tau$ are two flexible parameters. Changing $s$, $\tau$ can derivate different wavelet functions. Changing $s$ can make wave shape of function stretch and compress along time axis. Simultaneously, if the corresponding frequency spectra of $W(t)$ is $\psi(\omega)$, then the corresponding frequency spectra of $W(t/s)$ is $\psi(s\omega)$, because:

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} W\left(\frac{t}{s}\right) e^{-i\omega t} dt = \frac{s}{2\pi} \int_{-\infty}^{\infty} W\left(\frac{t}{s}\right) e^{-is\omega t} dt = s \cdot \psi(s\omega)$$

(2)

so changing $s$ simultaneously changes analytic frequency channel, while changing $\tau$ cause waveform of function to move along time axis.

Among the wavelet function family, several wavelet functions are verified very useful in practice. We can understand main characteristics of these wavelet functions in the wavelet toolbox of MATLAB. The frequently used wavelet functions are Haar wavelet, Daubechies (simply dbN) wavelet series, Biorthogonal (simply biorNr.Nd) wavelet series, Coiflet (simply coifN) wavelet series, SymletsA (simply symN) wavelet series, Morlet (simply morl) wavelet, Mexican Hat (simply mexh) wavelet, Meyer function and Battle-Lemorize wavelet.

1.2 Wavelet transform

It has been discussed above that changing $s$, $\tau$ can derivate different wavelet functions. Changing $s$ can make wave shape of function stretch and compress along time axis and form different order wavelet. Changing $\tau$ can make function move along time axis and form different position wavelet. Make wavelet transform to signal $f(t)$ by using the following wavelet function:

$$a(s,\tau) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} f(t) W_{s,\tau}^{*}(t) dt$$

(3)

where $W_{s,\tau}^{*}(t)$ is the conjugate function of $W_{s,\tau}(t)$. To do inverse transform to equation (3), we obtain

$$f(t) = \frac{1}{C_s} \int_{-\infty}^{\infty} f(s,\tau) W_{s,\tau}(t) \frac{dsd\tau}{s^2}$$

(4)