Application of Full-Order and Simplified EKFs to Sensorless PM Brushless AC Machines

Xi Zhu, Zi-Qiang Zhu*, David Howe
Department of Electronic and Electrical Engineering, University of Sheffield, Sheffield S1 3JD, UK

Abstract: This paper employs an extended Kalman filter (EKF) to estimate the rotor position and speed of a vector controlled surface-mounted permanent magnet (PM) brushless AC (BLAC) motor from measured terminal voltages and currents only. Both full-order and simplified EKFs are employed and their simulated performance capabilities are compared. Excellent agreement is achieved between estimated and commanded results. The EKF is also employed to identify the stator flux-linkage due to the PMs, which is influenced by temperature variation and magnetic saturation.

Keywords: Brushless AC drives, extended Kalman filter, parameter identification, permanent magnet, sensorless.

1 Introduction

Due to their high power density and efficiency, permanent magnet (PM) brushless AC (BLAC) drives are now employed in a wide range of applications from high performance servo drives to aerospace aircraft flight control surface actuators. A continuing reduction in the price of high-energy PMs, in particular neodymium-iron-boron, will further accelerate the uptake of PM brushless machine technology in variable-speed applications.

The sensorless control of PM BLAC drives, in which the rotor position and speed are estimated mathematically instead of by discrete sensors, has also received considerable attention since it improves reliability in addition to reducing the cost and size of drive systems. Thus, significant effort has been focused on identifying the states of a BLAC drive for sensorless control.

The Kalman filter, introduced by R. Kalman in 1960 and subsequently refined by Kalman and R. Bucy, is an optimal recursive algorithm which can tolerate measurement error (measurement noise) as well as inaccuracy in mathematical models caused by variations in parameters or non-modeled dynamics (process noise). It has been used in a wide variety of applications, such as navigation, industrial process control, tracking, system identification, etc. However, it is only relatively recently that Kalman filter techniques have been employed as state and parameter observers in high performance BLAC drive systems[1-7].

In this paper, both full-order extended Kalman filter (EKF) and simplified EKF models are developed to estimate the rotor speed and position of a surface-mounted magnet BLAC motor. Excellent sensorless performance is achieved without the phase delay which normally results with other sensorless control methods when only stator currents and voltages are measured. Thus, it is potentially more attractive. In addition, the EKF is employed to identify the stator flux-linkage due to PMs.

2 Mathematical model of a BLAC motor

The mathematical model for a surface-mounted magnet BLAC motor in the rotor d-q reference frame can be expressed as:

\[
\begin{align*}
\mathbf{u}_d &= r\mathbf{i}_d + \frac{d\lambda_d}{dt} - \omega_r\lambda_q \\
\mathbf{u}_q &= r\mathbf{i}_q + \frac{d\lambda_q}{dt} + \omega_r\lambda_d
\end{align*}
\]  

(1)

where \(\omega_r\) is the electrical angular speed of the rotor, \(r\) is stator phase resistance, \(p\) is the number of pole-pairs, and \(\lambda_{d-q}, u_{d-q}, i_{d-q}\) and \(L_{d-q}\), are the \(d\)-axis and \(q\)-axis PM flux-linkages, voltages, currents, and inductances, respectively. For a surface-mounted magnet BLAC motor, \(L_d = L_q = L\), and

\[
\begin{align*}
\lambda_d &= L\mathbf{i}_d + \lambda_m \\
\lambda_q &= L\mathbf{i}_q
\end{align*}
\]  

(2)

The electromagnetic torque in the rotor reference frame can be expressed as:

\[
T_e = \frac{3}{2}p\lambda_m\mathbf{i}_q
\]  

(3)
while the Park transformation is
\[
\begin{bmatrix}
x_d \\
x_q \\
x_α \\
x_β
\end{bmatrix} =
\begin{bmatrix}
\cos θ_r & \sin θ_r \\
-\sin θ_r & \cos θ_r \\
\cos θ_r & -\sin θ_r \\
\sin θ_r & \cos θ_r
\end{bmatrix}
\begin{bmatrix}
x_α \\
x_β \\
x_q \\
x_d
\end{bmatrix}
\]
\tag{4}
\]
\[
\begin{bmatrix}
x_α \\
x_β
\end{bmatrix} =
\begin{bmatrix}
1 & -\frac{1}{2} & \frac{1}{2} \\
0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}
\end{bmatrix}
\begin{bmatrix}
x_α \\
x_β \\
x_q
\end{bmatrix}
\]
\tag{5}
\]

where \( x \) can be the voltage, current or flux-linkage. Thus, three-phase stator currents can be transformed to the \( d-q \) axis rotor frame, and the current vector can be decomposed into \( d \)- and \( q \)-axis components. For the Park transformation, the \( d \)-axis is defined as the axis of PM flux (i.e. the rotor pole axis), and the \( q \)-axis is in quadrature with the \( d \)-axis. Thus, in the stationary \( α-β \) reference frame, the model can be expressed as:
\[
\begin{aligned}
u_α &= r_iα + \frac{dλ_α}{dt} \\
u_β &= r_iβ + \frac{dλ_β}{dt}
\end{aligned}
\]
\tag{7}
\]

where
\[
\begin{bmatrix}
λ_α \\
λ_β
\end{bmatrix} =
\begin{bmatrix}
\cos θ_r & -\sin θ_r \\
\sin θ_r & \cos θ_r
\end{bmatrix}
\begin{bmatrix}
λ_d \\
λ_q
\end{bmatrix}
\]
\tag{8}
\]

and the electromagnetic torque is given by
\[
T_e = \frac{3}{2} p(λ_α i_β - λ_β i_α).
\tag{9}
\]

The equation which governs the mechanical system is given by
\[
T_e = T_L + \frac{J}{p} \frac{dω_r}{dt} + \frac{D}{p} ω_r
\tag{10}
\]

where \( T_L \) is load torque, \( J \) is rotor inertia, and \( D \) is the viscous-friction damping coefficient. Thus,
\[
\begin{aligned}
\frac{dω_r}{dt} &= \frac{pT_e}{J} - \frac{Dω_r}{J} + \frac{pT_L}{J} \\
\frac{dθ_r}{dt} &= ω_r
\end{aligned}
\tag{11}
\]

A schematic of a basic vector controlled PM BLAC drive system which includes a PI speed controller, a current controller, a Park transformation, a power electronic inverter, and a PM BLAC motor is shown in Fig.1.

The current reference \( I^* \) for the torque command is obtained from the speed controller. For maximum torque per ampere, the \( d \)-axis current reference \( I_d^* \) is zero, and the \( q \)-axis current reference \( I_q^* \) is equal to the current reference \( I^* \). The three-phase current references \( I_{d}^*, I_{β}^*, \) and \( I_c^* \) are obtained from the \( d-q \) axis current references \( I_d^* \) and \( I_q^* \) via a \( d-q/abc \) transformation. The actual phase currents \( I_a, I_b, \) and \( I_c \) are regulated by a hysteresis current controller to follow the current references \( I_{d}^*, I_{β}^*, \) and \( I_c^* \).

3 EKF models

A. A full-order EKF model

The EKF algorithm\cite{1,3,4} is described in detail in the Appendix.

In order to estimate rotor speed and position, a full order EKF model was proposed in \cite{1}, although it is relatively computationally intensive. The differential equations for a BLAC drive are as follows:
\[
\begin{aligned}
di_α &= -\frac{r}{L} i_α + \frac{λ_m}{L} ω_r \sin θ_r + \frac{u_α}{L} \\
di_β &= -\frac{r}{L} i_β - \frac{λ_m}{L} ω_r \cos θ_r + \frac{u_β}{L} \\
dω_r &= \frac{3p^2}{2J} λ_m (-i_α \sin θ_r + i_β \cos θ_r) - \frac{B}{J} ω_r - \frac{P}{J} T_L
\end{aligned}
\tag{12}
\]

The linearized system is obtained as:
\[
\Gamma_k = \frac{∂f(x,k)}{∂x} \bigg|_{x=x_k}
\]
\[
= I +
\begin{bmatrix}
-r & \frac{λ_m}{L} \sin θ_r & \frac{ω_r}{L} λ_m \sin θ_r \\
\frac{r}{L} & 0 & \frac{ω_r}{L} λ_m \sin θ_r \\
\frac{u_α}{L} & \frac{u_β}{L} & 0
\end{bmatrix}
\begin{bmatrix}
S_1 \\
S_2 \\
S_3
\end{bmatrix}
\tag{13}
\]

where
\[
\begin{aligned}
S_1 &= -\frac{3p^2}{2J} λ_m \sin θ_r \\
S_2 &= \frac{3p^2}{2J} λ_m \cos θ_r
\end{aligned}
\]