Interpretation of Flow Instability Using Dynamic Material Modeling

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Dynamic material modeling (DMM)\textsuperscript{[1\textendash}7\textsuperscript{]} aims to correlate the constitutive behavior with microstructural evolution, flow instability and hot workability. This approach applies some of the principles of irreversible thermodynamics to the continuum mechanics of large plastic flow. The model uses the concepts of systems engineering. The workpiece undergoing hot working is considered to be a nonlinear dissipator of power and its constitutive behavior describes the manner in which power is dissipated to the surroundings. The total power dissipated by the workpiece per unit volume, \( P \), is given by

\[
P = \sigma \varepsilon
\]  \hspace{1cm} \text{(1)}

where \( \sigma \) and \( \varepsilon \) are the flow stress and the strain rate, respectively. The area under the stress-strain rate curve (\( G \)) has been termed as the dissipator content and is given by

\[
G = \int_0^\varepsilon \sigma \, d\varepsilon
\]  \hspace{1cm} \text{(2)}

According to DMM, this component is directly dissipated as heat. The complementary part of \( G \), termed as the dissipator co-content (\( J \)), is given by

\[
J = \int_0^\varepsilon \varepsilon \, d\sigma
\]  \hspace{1cm} \text{(3)}

The co-content (\( J \)) is first used in effecting microstructural transformations and subsequently released as heat. When the workpiece material follows the constitutive equation

\[
\sigma = K (\varepsilon)^m
\]  \hspace{1cm} \text{(4)}

where \( m \) is the strain rate sensitivity and \( K \) is a constant, the dissipator co-content will be given by

\[
J = \frac{\sigma m \varepsilon}{m + 1}
\]  \hspace{1cm} \text{(5)}

At one extreme, \( J \) can be as high as \( G \), when \( m = 1 \); and, on the other hand, \( J = 0 \), when \( m = 0 \). The case of \( m = 1 \) has been considered to be the ideal case, and an efficiency term has been defined as

\[
\eta = \frac{J}{J_{\text{max}}} = \frac{2m}{m + 1}
\]  \hspace{1cm} \text{(6)}

Murty \textit{et al.}\textsuperscript{[8]} have extended the calculation of \( \eta \) for the general case where Eq. (4) is not obeyed over a large range of strain rates.

Kumar\textsuperscript{[9]} and Alexander\textsuperscript{[4]} have extended DMM for analysis of flow instabilities during hot working. Kumar integrated DMM with Ziegler’s instability criterion to arrive at an instability criterion. Similarly, Gegel \textit{et al.} and Alexander integrated DMM with Lyapunov’s stability theory to arrive at an instability criterion.

Although DMM and its extensions have been applied to a large number of alloys\textsuperscript{[2\textendash}6\textsuperscript{]} \textit{recently}, the physical interpretation of \( G, J, \) and \( \eta \) has been re-examined by Montheillet \textit{et al.}\textsuperscript{[10]} and Ghosh.\textsuperscript{[11]} Montheillet \textit{et al.} have shown that \( G \) and \( J \) cannot be given interpretations analogous to that of \( U \) (strain energy density) and \( V \) (complementary energy density). Ghosh has shown with help of a few examples that the parameters \( J \) and \( \eta \) do not provide any information on partitioning of energy dissipation between dissipation through storage (microstructural mechanism) and thermal dissipation. The present article critically analyzes the interpretation of flow instability using DMM, as suggested by Kumar\textsuperscript{[9]} as well as by Gegel \textit{et al.}\textsuperscript{[2,3]} and Alexander.\textsuperscript{[4]}

Kumar\textsuperscript{[9]} has integrated Ziegler’s instability criterion with the DMM. According to Ziegler,\textsuperscript{[12]} a system undergoing large plastic deformation will be unstable if

\[
\frac{dD(\varepsilon)}{d\varepsilon} < \frac{D(\varepsilon)}{\varepsilon}
\]  \hspace{1cm} \text{(7)}

where \( D \) is given by\textsuperscript{[13]}

\[
D = \sigma \varepsilon - \frac{\partial G}{\partial \varepsilon} \varepsilon = T \frac{dS_p}{dt}
\]  \hspace{1cm} \text{(8)}

Here \( G \) is the free energy and \( (dS_p/dt) \) is rate of entropy generation in the system. Thus, \( D \) denotes the difference
between the applied power and the rate of change of Gibb’s free energy with time.

Kumar[9] extended Ziegler’s criterion by assuming that \( D = J \) and thus arrived at

\[
\xi = \frac{\partial \ln \left( \frac{m}{m+1} \right)}{\partial \ln \varepsilon} + m < 0 \tag{9}
\]

as the criterion for flow instability. There are three major difficulties with the extension of Ziegler’s criterion by Kumar, which are as follows.

(1) The term \( D \), which is given by Eq. [8], cannot be substituted by the power co-content \( J \). Let us consider the case of high-temperature steady-state (constant flow stress) plastic deformation at very low strain rates. Constant flow stress implies constant dislocation density. Therefore, in this case, there is no change in the stored elastic energy (which includes the elastic energy due to the presence of dislocations) during the course of deformation. Moreover, at very low strain rates, plastic deformation does not lead to a significant rise in temperature. Therefore, the rate of change of free energy \( G^f \) with time (or strain) can be neglected. For this condition, \( D \) therefore is the rate of energy dissipation as heat, which is equal to the total power applied. Since, for this particular case, \( D \) is not equal to \( J \), we can conclude that \( "D = J" \) is not always valid.

Also, it can be seen from Eq. [8] that Ziegler[13] has taken into account the variation of \( G^f \) with strain (time). Only in certain cases, like that of steady-state deformation at very low strain rates, will \( G^f \) remain constant with time. Otherwise, it will change with time and its rate of change with respect to time has to be incorporated into the expression for \( D \), as per Eq. [8]. In other words, the change in stress, defect density, and other variables (that determine the free energy) with strain have to be taken into account. However, in Kumar’s approach, evolution with strain plays no role in predicting flow instability.

(2) Kumar’s criterion may incorrectly predict flow instability in the transition between regions II and III of structural superplasticity. Structurally superplastic materials usually exhibit a sigmoidal relationship between the steady-state flow stress and strain rate, when the data are plotted logarithmically. As per this variation, the strain rate domain is divided into three regimes, commonly referred to as regions I, II, and III of superplasticity. The intermediate regime of strain rate, i.e., region II, exhibits higher strain rate sensitivities \( (0.4 < m < 1.0) \) than the low (region I) and high (region III) strain rate regimes. Now, let us analyze the consequences of this variation in the light of Kumar’s criterion.

By applying the chain rule, Inequality [9] can be rewritten as

\[
\xi = \frac{m}{m+1} \frac{\partial}{\partial m} \ln \left( \frac{m}{m+1} \right) \frac{\partial m}{\partial \ln \varepsilon} + m < 0
\]

or

\[
\xi = \frac{1}{m(m+1)} \frac{\partial m}{\partial \ln \varepsilon} + m < 0 \tag{10}
\]

From the preceding expression, one can see that Kumar’s criterion predicts flow instability when the rate of change of strain rate sensitivity with respect to the logarithm of the strain rate is less than a critical value, i.e., \(-2.3 \, m^2 \) \((m + 1)\), which is a function of \(m\). In the case of structural superplasticity, it is possible for Kumar’s criterion to incorrectly predict flow instability in the domain lying in the transition from region II to region III of superplasticity. This is because of the highly negative value of \( (\partial m/\partial \log \varepsilon) \) in this transition regime, which eliminates the requirement for a very low value of \( m \) in order for the condition \( \xi < 0 \) to be satisfied.

If we analyze data published by Murty et al.[14] on rapidly solidified/mechanically alloyed Al-Ti–alloys (AT-4 alloy, temperature = 575 °C, and strain rate range: 10^{-2} \, s^{-1} to 10^{-1} \, s^{-1}], Murty and Koczak[15] on Al-4 wt pct Ti alloy processed by a powder metallurgy route (grain size = 2.6 μm, temperature = 525 °C to 575 °C, and strain rate range: 10^{-2} \, s^{-1} to 10^{-1} \, s^{-1}], Agarwal et al.[16] on Ti-6Al-4V-0.018 Y_2O_3 (temperature = 927 °C, and strain rate range: 5 \times 10^{-4} \, s^{-1} to 5 \times 10^{-3} \, s^{-1}], and Ha et al.[17] on Zn-0.3 wt pct Al alloy[17] (room temperature, and strain rate range: 5 \times 10^{-3} \, s^{-1} to 5 \times 10^{-2} \, s^{-1}], we get flow instability as per Kumar’s criterion. In this range of strain rate, superplastic alloys are not known to exhibit flow instability at low strain or exhibit poor workability. In fact, in some of the aforementioned cases, the alloys exhibited high ductility.[14,17] Thus, Kumar’s criterion incorrectly predicts poor workability for many superplastic alloys in temperature-strain rate domains where the ductility is good.

(3) Kumar’s criterion rarely predicts inhomogeneous flow at low strain rates \( (\approx 0.1 \, s^{-1}) \). One of the manifestations of flow instability, called inhomogeneous flow, is observed at low strain rates \( (\approx 0.1 \, s^{-1}) \) in many alloys. At low strain rates, these alloys exhibit flow softening due to microstructural changes and the resultant inhomogeneous flow is successfully predicted by a flow localization parameter that takes work hardening (or work softening) into account, i.e., \( (\gamma - 1)/m \)[18] However, Kumar’s criterion rarely predicts flow instability at low strain rates \( (\approx 0.1 \, s^{-1}) \).

Although the extension of DMM by Kumar lacks rigour and has poor predictive ability at low strain rate, it has successfully predicted shear banding at high strain rates of 10 and 100 \, s^{-1} in a large number of alloys.[6] This can be understood if the highly negative value of \( \xi \) in the high strain rate domain of shear banding is taken to be merely a mathematical consequence of the very low value of strain rate sensitivity \( m \). Along with the obvious adiabatic flow softening in this regime, the low value of strain rate sensitivity is the actual cause of shear banding in these cases.

As per inequality [10], \( \xi \) will be highly negative whenever \( (\partial m/\partial \log \varepsilon) < 0 \) and the magnitude of \( (\partial m/\partial \log \varepsilon) \)