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Stability of an annular viscous liquid jet in compressible gases with different properties inside and outside of the jet

Abstract  A spatial linear instability analysis is conducted on an annular viscous liquid jet injected into compressible gases and a three-dimensional model of the jet is developed. The model takes into account differences between the velocities, densities of the gases inside and outside of the liquid jet. Theoretical analysis reveals that there exist 9 dimensionless parameters controlling the instability of the liquid jet. Numerical computations reveal some basic characteristics in the breakup and atomization process of the liquid jet as well as influences of these relevant parameters. Major observations and findings of this study are as follows. The Mach number plays a destabilizing role and the inner Mach number has a greater effect on the jet instability than the outer Mach number. The Reynolds number always tends to promote the instabilities of the liquid jet, but its influence is very limited. The Weber number and the gas-to-liquid density ratio also have unstable effects and can improve the atomization of liquid jets. Furthermore, the effects of the Weber number and gas-to-liquid density ratio on the maximum growth rates of axisymmetric and non-axisymmetric disturbances and corresponding dominant wave numbers are manifested in a linear way, while that of the Mach number is non-linear. The effect of Reynolds on the maximum growth rates is non-linear, but the dominant wavenumber is almost not affected by the Reynolds number.

Keywords  liquid jet, dominant wave number, compressibility, instability

1 Introduction

On account of its theoretical importance and wide applications in engineering, the breakup and atomization of liquid jets has been drawing more and more interest in recent years. Theoretical studies on the mechanism of the breakup and atomization of liquid jets have been mainly based on the hydrodynamic instability theory, according to which the basic mechanism of the jet breakup and atomization is attributed to the interaction between the high-speed jet and the surrounding air or gas, which causes the unstable wave on the liquid-gas interface to increase quickly and selectively. Up to now, research in this area has been focused mainly on the liquid jet injected into incompressible gases [1–8]. However, at higher relative velocities between liquid and gas, the effects of gas compressibility on jet breakup and atomization are important and non-negligible. Presently, little study on the breakup and atomization of liquid jets in compressible gases can be found. Zhou and Lin [9] presented a 3-D model for the breakup process of a cylindrical jet, but their stability analysis was performed on a 2-D basis, and the effect of liquid viscosity was not taken into account. Ma and Zhou [10] developed a 2-D model for a cylindrical jet of viscous liquid in high speed inviscid compressible gases, and conducted a stability analysis of the gas-liquid interface based on the temporal mode. Subramaniam et al. [11] investigated the temporal instability of a cylindrical gas jet in a quiescent liquid. Yan and Xie [12] developed a 3-D mathematical model for an annular viscous liquid jet injected into a compressible gas, and conducted an analysis of its spatial stability. The present work extends the analysis of Ref. [12] to a more general situation in such a way that differences between velocities and densities of the gases inside and outside of the liquid jet are taken into account and a solution to this comprehensive model is obtained.
2 Stability analysis of liquid jet

2.1 Solution to governing equations of jet perturbation

Considering that an incompressible liquid of density $\rho_l$ is injected from an annular nozzle into an inviscid and compressible moving gas, assuming that the direction of the liquid jet and the gas is opposite the direction of the $z$-axes in cylindrical coordinates of $(r, \theta, z)$, the inner and outer radii of the annular nozzle are $a$ and $b$ respectively, the thickness of the liquid jet is $h = b - a$, the densities of the inner and outer gases are $\rho_{2i}$, $\rho_{2o}$ respectively, the kinematic viscosity of the liquid is $\nu_l$, and the surface tensions at the liquid-gas interfaces is $\sigma$, the basic state of the jet system is given as:

The liquid velocity: $\bar{U}_1 = (0, 0, -U_1)$, the velocity of the gas inside of the liquid jet $\bar{U}_2 = (0, 0, -U_{2i})$, and the velocity of the gas outside of the jet: $\bar{U}_2 = (0, 0, -U_{2o})$, $\bar{p}_1 - \bar{p}_{2i} = \frac{\sigma}{a}$, $\bar{p}_1 - \bar{p}_{2o} = \frac{\sigma}{b}$, where $\bar{p}_{2i}$, $\bar{p}_{2o}$ are the gas pressures of the inside and outside of the intact cylindrical liquid jet.

Perturbing and linearizing the governing equations, i.e., the N-S equations, the linearized dimensionless perturbation equations can be obtained as follows:

$$\frac{\partial \tilde{p}_j}{\partial t} + Q_j \nabla \cdot V_j = 0, \tag{1}$$

$$\frac{\partial V_j}{\partial t} - \delta_{j} N_j \frac{\partial V_j}{\partial z} = -\frac{1}{Q_j} \nabla \bar{p}_j + \frac{\delta_{j}}{Re} \nabla^2 V_j. \tag{2}$$

Furthermore, assuming that the gaseous media are ideal gas and the disturbance propagation in the gas is a reversible adiabatic process, the following dimensionless relation is obtained:

For the gas inside the jet

$$\frac{\partial \tilde{p}_2}{\partial \bar{p}_2} = \frac{1}{Ma_i^2}. \tag{3}$$

For the gas outside of the jet

$$\frac{\partial \tilde{p}_2}{\partial \bar{p}_2} = \frac{1}{Ma_o^2}. \tag{4}$$

where, the subscript $j = 1, 2$ referring to the liquid and the gas phases; $V_j$ and $p_j$ are the disturbance velocity and pressure.

For the gas phase: $j = 1$, $\delta_{21} = 0$, $\delta_{11} = 1$,

$$Q_1 = \frac{\bar{p}_1}{\rho_1} = 1, N_1 = \frac{U_1}{U_1} = 1,$$

for the liquid phase:

$$j = 2$$

$$\delta_{22} = 1, \delta_{12} = 0$$

for the gas phase. For the inner gas

$$Q_2 = Q_i = \frac{\bar{p}_{2i}}{\bar{p}_1} N_2 = N_i = \frac{U_{2i}}{U_1}, Ma = Ma_i = \frac{U_1}{c_{2i}},$$

and for the outer gas

$$Q_2 = Q_o = \frac{\bar{p}_{2o}}{\bar{p}_1} N_2 = N_o = \frac{U_{2o}}{U_1}, Ma = Ma_o = \frac{U_1}{c_{2o}},$$

where, $Q_i$, $Q_o$ are the inner and outer gas-to-liquid density ratios; $N_i$, $N_o$ gas-to-liquid velocity ratios; and $Ma_i$, $Ma_o$ the Mach numbers related to the liquid jet coordinate system; $c_{2i}$ and $c_{2o}$ are the sound speeds in the inner and outer gases, respectively; $Re = Uh/v$ is Reynolds number.

Obviously, under the conditions of $Q_i = Q_o$, $N_i = N_o = 0$, $Ma_i = Ma_o$, Equations (1), (2), (3) and (4) are reduced to the case addressed in Ref. [12].

To solve the perturbation equations, a normal mode method is adopted, assuming that the disturbance velocity and pressure $p_j$, $V_j$ have the form:

$$(p_j, V_j) = \left(\bar{p}_j V_j \right) \exp[\omega t + i(kz + m\theta)]$$

where $\bar{V}_j(r) = (V_{jr}(r), V_{j\theta}(r), V_{jz}(r))$, and $\omega = \omega_r + i\omega_i$ is the complex frequency, whose real part gives the temporal growth rate, the imaginary part gives the frequency; $k = k_z + ik_\theta$, where $k_z = 2\pi/\lambda_0$ is the wave number and $\lambda_0$ is wavelength, $k_\theta$ is the spatial amplification rate of the disturbance, the real number $m$ is called the azimuthal wave number or mode number, which characterizes the azimuthal deformation of the perturbed jet. If $m = 0$, the disturbance is axisymmetrical, and it is asymmetrical if $m \geq 1$.

Solving the perturbation equations for the liquid phase gives liquid velocities and pressure, which are identical with those presented in Ref. [12]. And from the solution to the perturbation equations for the gas phase, the following equations can be obtained:

$$V_{r2} = \frac{s}{Q(\omega - iNk)} \cdot \left[ d_{21} I_m'(sr) + d_{22} K_m'(sr) \right] \cdot \exp[\omega t + i(kz + m\theta)], \tag{5}$$

$$V_{\theta2} = \frac{im}{Q(\omega - iNk)r} \cdot \left[ d_{21} I_m(sr) + d_{22} K_m(sr) \right] \cdot \exp[\omega t + i(kz + m\theta)], \tag{6}$$

$$V_{z2} = \frac{ik}{Q(\omega - iNk)} \cdot \left[ d_{21} I_m(sr) + d_{22} K_m(sr) \right] \cdot \exp[\omega t + i(kz + m\theta)], \tag{7}$$

$$p_2 = [d_{21} I_m(kr) + d_{22} K_m(kr)]\exp[\omega t + i(kz + m\theta)], \tag{8}$$

where