Determination of mechanical parameters for elements in meso-mechanical models of concrete

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ABSTRACT The responses of cement mortar specimens of different dimensions under compression and tension were calculated based on the discrete element method with the modified-rigid-body-spring concrete model, in which the mechanical parameters derived from macro-scale material tests were applied directly to the mortar elements. By comparing the calculated results with those predicted by the Carpinteri and Weibull size effects laws, a series of formulas to convert the macro-scale mechanical parameters of mortar and interface to those at the meso-scale were proposed through a fitting analysis. Based on the proposed formulas, numerical simulation of axial compressive and tensile failure processes of concrete and cement mortar materials, respectively were conducted. The calculated results were a good match with the test results.

KEYWORDS concrete, meso-mechanical model, discrete element method, size effect, mechanical parameter

1 Introduction

Concrete is a heterogeneous material that consists of mortar, coarse aggregates and a meso-level interfacial transition zone. It is known that the mechanical behavior and failure modes of concrete at the macro-scale are strongly affected by its meso-structures [1]. Therefore, an approach to research the failure process of concrete in the meso-scale is significant for a further understanding of the failure mechanism, strength and deformation characteristics of concrete at the macro-scale.

When the mechanical properties of concrete are analyzed using meso-scale numerical methods, key points are the establishment of rational meso-mechanical models and an approach to determine the material parameters for elements in the models. The meso-mechanic models of concrete can be divided into two categories according to the assumptions of materials [2]. Finite element method (FEM) based models, such as microplane model [3], lattice model [4] and random particle model [5], belong to the first category, in which the material is assumed to be continuous. Discrete element method (DEM) based models, such as the modified-rigid-body-spring model [6], a beam-particle model [7] and the extended distinct element method [8], are classified in the second category, in which the continuity assumption is not needed.

In FEM-based models, the boundaries between adjacent elements need to meet the requirement of deformation compatibility, making it difficult to simulate the cracking process in concrete. Moreover, a numerical solution may become unstable when the behavior of concrete is highly nonlinear. In DEM-based models, these shortcomings can be easily overcome, because the adjacent elements can be separated in the analysis of the failure process of concrete.

In the application of FEM- and DEM-based models, the mechanical parameters of elements, such as the strength of mortar and the interface elements at a meso-scale, first need to be established. Theoretically, the mechanical parameters of elements should be determined through a meso-scale material test. However, it is difficult to carry out that kind of the test, due to the limitations of the current testing technology. The most applicable way is the conversion of the mechanical parameters obtained from traditional macro-scale material tests. However, the question is how to convert the mechanical parameters of materials from the macro-scale to the meso-scale.

Nagai et al. carried out simulations on failure processes of mortar and concrete with two-dimensional (2D) and
three-dimensional (3D) rigid-body-spring-models (RBSMs) and concluded that, due to the different dimensions of the elements, the values of the material property of elements at the meso-scale were different from those of the analyzed object in the macro-scale [9,10]. Accordingly, relationships between Poisson’s ratio $\nu_{\text{elem}}$ and elastic modulus $E_{\text{elem}}$ of elements in the meso-scale and Poisson’s ratio $\nu$ and elastic modulus $E$ in the macro-scale were proposed, which were used in the 2D and 3D RBSMs.

Xing and Yu [11] presented empirical conversion formulas of mechanical parameters, including the strength and elastic modulus, from the macro-scale to the meso-scale by repeated calculation of the compressive concrete specimens using a beam-particle model.

The existing research results are good examples of the adjustment of elements’ mechanical parameter values in the meso-scale. However, the elements are different in different models. Generally, in DEM-based models, concrete is assumed to be a composite material of mortar, coarse aggregates and their interface. For normal concrete, the coarse aggregates usually do not fail when the concrete is subjected to compression or tension; and, the mechanical behavior of the interfaces is mainly affected by the mechanical behavior of the mortar and surface texture of the aggregates. Therefore, it is important to study the mechanical behavior of mortar at the meso-scale if the concrete material is modeled based on DEM.

In this study, we focus on the mechanical parameters of mortar elements in DEM-based models. First, the pure mortar specimens with different dimensions under compression and tension are calculated using the modified-rigid-body-spring concrete model proposed by the authors [6]. The calculated results are then compared with those predicted with the existing size effect laws. The calculated results were verified through a comparison with experimental results. Finally, a series of formulas for conversion of the mechanical parameters of mortar and interface in the macro-scale to the mesoscale are proposed, according to the calculation results for the mortar specimens and the Weibull size effect law. The proposed formulas are applied in the numerical analysis of concrete materials.

## 2 Modified-rigid-body-spring concrete model based on DEM

The authors developed a 2D modified-rigid-body-spring concrete model based on DEM [6], in which concrete is assumed to be a three-phase composite material, with coarse aggregates as the dispersed phase, mortar as the continuous phase, and zero-sized interfaces as the interfacial phase. The coarse aggregates in a concrete section were assumed to be circular. According to the Walraven function [12], the diameter and number of coarse aggregates in a cross section of two dimensional model can be calculated and the distribution probability of an aggregate with a certain diameter can be obtained from the Fuller’s gradation [13]. The probability of the diameter ($D$) of a circular aggregate particle being smaller than $D_i$ (where $i$ is the number of the grade and $j = 1, 2$ for the upper or lower bound of grade $i$, respectively) can be expressed as:

$$P_i(D<D_{\text{max}}) = P_k(1.065d^{1.5} - 0.053d^4 - 0.012d^6 - 0.0045d_8 + 0.0025d^{10}),$$

where $d = D_i/D_{\text{max}}; D_{\text{max}}$ is the maximum size of the aggregate particle; and, $P_k$ is the aggregate volume fraction.

According to the different phases, the element mesh is generated with a Voronoi diagram to reduce the influence of the mesh arrangement on the cracking direction. The generated element mesh for a cross section of concrete specimen is shown in Fig. 1.

![Fig. 1 Generated element mes](image)

Each element has one rotational and two transitional degrees of freedom. The polyhedron elements are interconnected by springs including normal and shear springs, as shown in Fig. 2, where $x_i$, $y_i$, $\theta_i$; and $x_j$, $y_j$, $\theta_j$ are the horizontal, vertical and rotational displacements of elements $i$ and $j$, respectively; $k_{n,s}, k_{s,s}$ and $\Delta_{n}, \Delta_{s}$ are the stiffness and deformation of the normal and shear springs, respectively; $l$ is the length of the common boundary of the two elements; and, $h_i$ and $h_j$ are the lengths of the perpendicular line from the center of gravity of elements $i$ and $j$ to the common boundary, respectively. There are also dampers with damping coefficients $c_n$ and $c_s$ along the boundary, which are used to dissipate energy when solving static problems using the dynamic relaxation method.

Two kinds of spring groups are defined to simulate the connection of mortar and the connection between mortar