The paper under discussion is a well-written expository review of the vast contribution of the author and his many collaborators to the stability theory for linear optimization problems of the form

$$P: \quad \inf_{x} \left\{ \langle c^*, x \rangle, \langle a_t^*, x \rangle \leq b_t, \quad t \in T \right\},$$

where $x$ ranges on a locally convex Hausdorff topological vector space $X$ whose topological dual $X^*$ contains $c^*$ and $a_t^*$, $t \in T$, whereas $b_t \in \mathbb{R}$, $t \in T$. The problem $P$ is called continuous whenever $T$ is a compact Hausdorff topological space and the functions $t \mapsto a_t^*$, with $X^*$ equipped with the weak* topology, and $t \mapsto b_t$ are continuous on $T$; $P$ is an ordinary linear programming (LP) problem when the dimension of $X$ and the cardinality of $T$ are finite, it is a linear infinite programming (LIP) problem when both are infinite, and it is a linear semi-infinite programming (LSIP) problem otherwise. Any LP problem $P$ can be seen as a continuous problem just taking the discrete topology on $T$, so that the stability theory for LP problems is subsumed by the one corresponding to continuous linear optimization problems. The first LP and LSIP problems were posed in the XVIII Century, for getting best approximate solutions of overdetermined linear systems arising in astronomy, and in 1939, in a pre doctoral work of G. Dantzig on statistical inference where he already used his geometry of columns (i.e., the basis of the simplex method), respectively. The oldest LIP model reported in the nice textbook of Anderson and Nash (1987) on the subject is the formulation of the mass-transfer problem by Kantorovich in 1942.


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My first comment concerns the opportunity of this review. In my opinion, the stability theory on linear optimization problems grew so much during the last 15 years that the researchers can hardly detect the open problems in this field whereas the practitioners could get lost when trying to perform stability analysis in practical applications without this kind of guided survey at hand. In fact, when dealing with linear optimization problems with uncertain data, after solving the nominal problem it is convenient to investigate the effect on the feasible set, the optimal set, and the optimal value of $P$, of small perturbations of the data. Sensitivity analysis allows to evaluate the impact on the optimal value of small perturbations of $P$ provided it is either an LP problem such that the function $t \mapsto a^*_t$ remains fixed or an LSIP problem satisfying very strong conditions. Otherwise, stability analysis is the unique available tool to carrying out post optimal analysis (besides simulation). Nevertheless, to the best of my knowledge, no LSIP application from those published in 2006–2010 performs any kind of post optimal analysis despite the obvious uncertainty of the data in most of the corresponding fields (Ozogur and Weber 2010 and Klabjan and Adelman 2007 also involve LP problems):

5. Optimal design (Karimi and Galdos 2010; Wan et al. 2009).
7. Other optimization problems (Bisbos and Ammpatzis 2008; Gómez and Gómez 2006; Krishnan and Mitchel 2006).

The conclusion is that the article under discussion will be a useful tool not only for researchers interested in stability in optimization, but also in order to approach this nice theory to the potential practitioners.

My second comment is intended to pay tribute to the pioneers in this field by means of a brief discussion of the antecedents, obviously precluded from the excellent survey under discussion for the sake of brevity (the oldest results reviewed in the paper are dated in 1996). The first works on stability in linear optimization were published by Daniel (1973, 1975) and Robinson (1975) in the mid-1970s, who analyzed the stability of the feasible set mapping $F$ from a quantitative perspective for linear programming problems and for linear infinite programming problems posed in Banach spaces, respectively. In the beginning of the 1980s deserve to be mentioned, together with the famous monograph of Bank et al. (1983), the works of Fischer (1983) and Brosowski (1984) on the stability properties of $F$, the optimal set mapping $F^*$ and the optimal value function $\vartheta$ for continuous LSIP problems (in terms of the author, the perturbation setting $C$). Similar problems and setting were considered by Todorov (1985–1986) from a generic point of view. On the occasion of the sabbatical year spent by the latter author at the University of Alicante in 1993 we