COEFFICIENT MULTIPLIERS ON MIXED NORM SPACES

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Abstract. This paper describes the characteristics of the coefficient multipliers of mixed norm spaces $A^{p,q}$ with $0 < p \leq 1, 0 < q, a < \infty$ into some analytic function spaces. In corollaries, the characteristics of the coefficient multipliers of $G^p (0 < p < 1)$ and $A^p (0 < p \leq 1)$ into some analytic function spaces are given.

§ 1 Introduction

Suppose that $f$ is analytic in the open unit disc $D$ in the complex plane. We define

$$M_p(r,f) = \left( \frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^p d\theta \right)^{1/p}, 0 < p < \infty;$$

$$M_\infty(r,f) = \max_{0 \leq \theta \leq 2\pi} |f(re^{i\theta})|;$$

$$H^p = \{ f : \| f \|_{H^p} = \sup_{0 < r < 1} M_p(r,f) < \infty \}, 0 < p \leq \infty;$$

$$G^p = \{ f : \| f \|_{G^p} = \left( \int_0^1 M_p(r,f)^p dr \right)^{1/p} < \infty \}, 0 < p < \infty;$$

$$B^p = \{ f : \| f \|_{B^p} = \int_0^1 (1 - r)^{1/p - 2} M_1(r,f) dr < \infty \}, 0 < p < 1;$$

$$A^p = \{ f : \| f \|_{A^p} = \left( \int_0^1 M_1^p(r,f) dr \right)^{1/p} < \infty \}, 0 < p < \infty; A^\infty = H^\infty;$$

$$A^{p,q,a} = \{ f : \| f \|_{A^{p,q,a}} = \left( \int_0^1 (1 - r)^{aq-1} M_p^q(r,f) dr \right)^{1/q} < \infty \},$$

$$0 < p \leq \infty, 0 < q, a < \infty.$$

These spaces form Banach spaces or Fréchet spaces. We refer to [1–4] for the properties of these spaces. $A^{p,q,a}$ are called mixed norm spaces. Obviously, $A^p = A^{p,p,1/p}$.

The collection of all coefficient multipliers of an analytic function space $A$ into $B$ is denoted by $(A,B)$. That is $(A,B) = \{ g : g * f \in B, \text{ whenever } f \in A \}$, where $g * f$ is Hadamard product of $g$ and $f$. Let $[x]$ denote the greatest integer not greater than $x$, $C$ denote a positive constant depending only on indices $p, q, a, \ldots$, or parameters of an argument. It may differ at different occurrences even in the same formula.

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The multipliers for $G^p (1 \leq p < \infty)$ spaces were studied in [5]. In this paper, we describe the characteristics of the coefficient multipliers of mixed norm spaces $A^{p,q,a} (0 < p \leq 1, 0 < q, a < \infty)$ into $H^t, A^t, G^t$ and $B^t$. As the corollaries of the results on the multipliers of $A^{p,q,a}$, we give the the characteristics of the coefficient multipliers of $G^p (0 < p < 1)$ and $A^p (0 < p \leq 1)$ into $H^q (1 \leq q < \infty), A^q (1 \leq q \leq \infty), G^q (p \leq q < \infty)$ and $B^q (0 < q < 1).

§ 2 Lemmas

Lemma 2.1. If $0 < p \leq \infty, 0 < q, a < \infty$, then $f \in A^{p,q,a}$ if and only if for any $s$ and $t$ with $p \leq s < \infty, q \leq t < \infty$,

$$\int_0^1 (1 - r)^{p(1/p - 1)} M_i (r, f) dr < \infty.$$ (2.1)

Proof. If $f \in A^{p,q,a}$, then $M_i (r, f) \leq C(1 - r)^{-a}$. Hence by Theorem 5.9 of [2],

$$\int_0^1 (1 - r)^{p(1/p - 1)} M_i (r, f) dr \leq C \int_0^1 (1 - r)^{p-1} M_i (r, f) dr = C \int_0^1 (1 - r)^{p-1} M_i (r, f) dr < \infty.$$ Conversely, if (2.1) holds, then letting $s = p$ and $t = q$, we obtain

$$\int_0^1 (1 - r)^{s-1} M_i (r, f) dr < \infty.$$ This implies $f \in A^{p,q,a}$.

Lemma 2.2. Suppose $0 < t \leq \infty, s = \min \{t, 1\}$. If $f$ and $g$ are analytic in $D, h = g * f$, then for any positive integer $m$,

$$r^m M_i (r^t, h^{(m)} (rz)) \leq C (1 - r)^{1-s/m} M_i (r, f) M_i (r, g^{(m)}).$$ (2.2)

Proof. By Parseval's formula, for $0 < r < 1$,

$$h (rz) = \frac{1}{2\pi} \int_0^{2\pi} f(re^{i\theta}) g (ze^{-i\theta}) d\phi.$$ Differentiating with respect to $z$ gives

$$r^m h^{(m)} (rz) = \frac{1}{2\pi} \int_0^{2\pi} f(re^{i\theta}) g^{(m)} (ze^{-i\theta}) e^{-im\theta} d\phi.$$ Hence

$$r^m \left| h^{(m)} (rz) \right| \leq \frac{1}{2\pi} \int_0^{2\pi} \left| f(re^{i\theta}) g^{(m)} (ze^{-i\theta}) \right| d\phi.$$ (2.3)

For $1 \leq t \leq \infty$, it follows from Jensen's inequality that

$$r^M_i (r^t, h^{(m)}) \leq M_i (r, f) M_i (r, g^{(m)}).$$ (2.4)

In the case of $t < 1$, by (2.3),

$$r^m \left| h^{(m)} (r^t e^{i\theta}) \right| \leq \frac{1}{2\pi} \int_0^{2\pi} \left| f(re^{i\theta}) \frac{g^{(m)} (re^{i\theta})}{g^{(m)} (re^{i\theta})} \right| d\phi.$$ Since for any $0 \leq \theta \leq 2\pi, f(re^{i\theta}) g^{(m)} (re^{i\theta})$ is analytic in $D$, it follows from Theorem 5.9 in [2] that