AN INEXACT LAGRANGE-NEWTON METHOD FOR STOCHASTIC QUADRATIC PROGRAMS WITH RECOURSE

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Abstract. In this paper, two-stage stochastic quadratic programming problems with equality constraints are considered. By Monte Carlo simulation-based approximations of the objective function and its first(second)derivative, an inexact Lagrange-Newton type method is proposed. It is showed that this method is globally convergent with probability one. In particular, the convergence is local superlinear under an integral approximation error bound condition. Moreover, this method can be easily extended to solve stochastic quadratic programming problems with inequality constraints.

§ 1 Introduction

In classical mathematical programming, parameters are assumed to be known exactly. For many practical optimization problems, however, one could have to take into account the uncertainty involved for a variety of reasons, for example, measurement errors, information about the future or unobserved events. Stochastic programming looks for some optimal decisions with taking this uncertainty into account. This field has received much attention and is developing rapidly with contributions from many disciplines such as operations research, probability and statistics, computer science, and economics. Many introductory books have been released in the last years, among them we mention in particular Birge and Louveaux\cite{1}, Kall and Wallace\cite{2}.

We consider the following two-stage stochastic quadratic programming problem.

\[ \min f(x) = \frac{1}{2} x^T G x + c^T x + Q(x), \]

subject to \( Ax = b, x \in \mathbb{R}^n \),

where

\[ Q(x) = \int_{\Omega} Q(x, \omega) p(\omega) d\omega, \]

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\[ Q(x, \omega) = \max \left\{ -\frac{1}{2} y^T Dy + y^T (\omega - Tx) \right\}, \]

subject to \( W y \leq q, y \in \mathbb{R}^n, \)

\( G \in \mathbb{R}^{nxn}, c \in \mathbb{R}^n, A \in \mathbb{R}^{nxn}, b \in \mathbb{R}^n, D \in \mathbb{R}^{nxn}, T \in \mathbb{R}^{nxn}, q \in \mathbb{R}^n, \) and \( W \in \mathbb{R}^{nxm} \) are fixed matrices or vectors; \( \omega \in \mathbb{R}^n \) is a random vector; \( p(\cdot) : \mathbb{R}^n \to \mathbb{R}^+ \) is a probability density function.

It is well known that the stochastic optimization problems are hard to solve because the objective function with uncertainty can be complicated and/or difficult to compute even approximately. To solve stochastic programming problems, one usually resorts to deterministic optimization methods. This idea is a natural one and was used by many authors over the years\([3-6]\). Deterministic methods were also applied to stochastic programming problems which involve quadratic programming in a vast literature. The extended linear-quadratic programming (ELQP) model was introduced by Rockafellar and Wets\([7-9]\). Qi\([10]\) proposed an SQP algorithm for ELQP problems. To solve (1.1), Chen, Qi and Womersley\([11]\) suggested a Newton-type approach, and showed that this method is globally convergent and locally superlinear convergent. At the same time, Birge, Chen and Qi\([12]\) investigated a stochastic Newton method for (1.1) with inequality constraints \( Ax \leq b \). Global convergence and local superlinear convergence of the method were established.

The paper addresses an inexact Lagrange-Newton-type approach to solve (1.1). Here we prove the global convergence of this method with probability one. Moreover, we also show the local superlinear convergence under the assumption that an integral approximation error bound condition holds. Finally, this method is extended to solve stochastic quadratic programming problems with inequality constraints.

The remainder of this paper is organized as follows. §2 gives the algorithm for (1.1). In §3 we discuss the convergence of the algorithm in detail. We proceed in §4 by showing the local superlinear convergence. In §5 we investigate the extension of problem (1.1). Finally, our conclusions are presented in §6.

§2 Inexact Lagrange-Newton-type algorithm

We assume that \( G \) is symmetric positive semi-definite, \( D \) is symmetric positive definite, \( p(\cdot) \) is continuously differentiable. Let \( X = \{ x \mid Ax = b, x \in \mathbb{R}^n \} \) and \( Y = \{ y \mid W y \leq q, y \in \mathbb{R}^n \} \) be nonempty polyhedral.

By Proposition 1 in [11], it is easy to see that the objective function \( f(x) \) in (1.1) is convex and has a continuous first-order derivative in \( \mathbb{R}^n \).

\[ g(x) := \nabla f(x) = Gx + c - T^T \int_{\mathbb{R}^n} f(\omega - Tx) p(\omega) d\omega, \]

where