AN EXTENSION OF CLARIAUT EQUATION
AND ITS APPLICATION

LIANG YOUDONG  ZOU ZHENQIANG  AND  WANG GUOZHAO

Abstract. The geodesic in differential geometry is commonly used in computer-aided filament winding (CAFW) to avoid slippage in manufacturing process. The uniqueness of the geodesic by its initial values severely restricts the choice of the fiber path and is an obstacle to the production of optimized structures. This paper presents a new class of more flexible non-slip trajectories on revolutional surfaces as an extension of the well-known Clariaut equation and gives its application in CAFW.

1. Introduction

Fiber composites represent a class of new materials providing high strength and stiffness together with light weight and excellent chemical resistance. Filament winding is a main production method allowing to fabricate composites with well-aligned fibers [8][12]. In early stage, geodesic winding technique was commonly used in composite material industry [7][9]. Although the geodesic is a basic concept in differential geometry [13], little attention of practical calculation has been paid by mathematicians. In many industries, geodesics are widely employed as fundamental techniques. For example, in ship-building industry, the geodesics are used to determine how a metal plate is bent to be a patch of the ship hull [14]. In American aerospace industry, people have known for a long time that the geodesic was a great design. During World War I, there were 10,000 bombers built with geodesic metal structures. It was the only airplane in continuous production throughout the entire length of World War I [11]. In composite material industry, the geodesic winding itself can not meet all the technical requirements such as turn-around and bridging-free. Modification and deviation from geodesics have always to be made in the design process. So, why not propose more flexible and general
winding technique to overcome these difficulties? — the main purpose of this paper \[2\][3][4].

Generally, the shapes of the filament wound parts are of tube-like or can be approximated as a closed revolutional surface. For a revolutional surface, the equation of the geodesic can be expressed by the well-known "Clariaut equation" \[1\] as follows

\[ rsina = \text{const}, \quad (1.1) \]

where \( r \) is a radius, \( a \) is the angle between the tangent and the central axis of the revolutional surface, and is called wind angle as shown in Fig. 1.

\[ \text{Fig. 1. A geodesic with wind angle } a \text{ on a revolutional surface} \]

Clariaut equation is simple and easy to calculate. In recent years many non-geodesics techniques have been proposed and developed \[5\][6][15]. Unfortunately, the differential equations of non-geodesics are more complicated and difficult to solve in a simple form. This paper presents an extension of the well-known Clariaut equation with the following advantages:

1. No slippage (stability),
2. Equation expressed in a single and analytical form (simplicity),
3. More freedom and easy to control (flexibility).

2. Semi-Geodesics

Recently, the semi-geodesic is one of the most common used techniques. Recall that the semi-geodesic is defined as the curve on a given surface \( S \) with constant slippage resistance \( \lambda \) \[5\]. Let \( r = r(s) \) be a fiber path \( C \) on \( S \), where \( s \) is the arc-length of \( C \). A trihedron consisting of three unit orthogonal vectors \( (t, n, b) \) at a point \( P \) on \( C \) can be determined; the tangent vector \( t \), normal vector \( n \) of \( S \) and \( b = t \times n \) as shown in Fig. 2.

The slippage resistance \( \lambda \) is defined as the following ratio

\[ \lambda = \pm \frac{|f_s|}{|f_n|}, \quad (2.1) \]