A GENERALIZATION OF RISK MODEL PERTURBED BY DIFFUSION

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Abstract. In this paper, the classical risk process perturbed by diffusion is generalized by allowing for "size fluctuation" and the ruin probability for this new model is discussed.

§ 1 Generalization of Classical Risk Process Perturbed by Diffusion

The classical risk process perturbed by diffusion (the following model (3)) was first introduced in [1] and further studied in [2] etc. Many results about ruin probability for this risk model have been gotten. The purpose of this paper is to generalize the classical risk process perturbed by diffusion by allowing for "size fluctuation" and discuss the ruin probability for the generalized risk model.

Let $(\Omega, \mathcal{F}, P)$ be a completed probability space containing all the objects defined in the following.

We now first introduce the concepts of random measure and Cox process.

Definition 1. If a stochastic process $\Lambda=\{\Lambda_t, t\geq 0\}$ satisfies the following conditions with probability 1,

(1) $\Lambda_0=0$,

(2) For each $t<-\infty, \Lambda_t<+\infty$,

(3) Its realization is the non-decreasing continuous function of time $t$,

then we call it a diffuse random measure.

Hereafter the random measure we meet is always assumed to be a diffuse random measure.

Definition 2. Let $\Lambda$ be a random measure, $\bar{N}=\{\bar{N}_t, t\geq 0\}$ be a standard Poisson process, $\Lambda$ and $\bar{N}$ are independent of each other. The point process $N=\bar{N} \ast \Lambda$ is called a Cox pro-
cess.

The classical risk process is

\[ R_t^0 = u + ct - \sum_{k=1}^{N_t} Z_k, \]  

(1)

where \( u \) denotes the initial capital, \( c \) the premium income; \( \{N_t, t \geq 0\} \) is a Poisson process with parameter \( \lambda > 0 \), it counts the number of the claims in the interval \( (0, t]\); \( \{Z_k, k \geq 1\} \) is a non-negative sequence of i.i.d. random variables, \( Z_k \) denotes the amount of the \( k \)th claim. \( R_t \) is the surplus of an insurance company at time \( t \); \( \{N_t\} \) and \( \{Z_k\} \) are independent of each other (see [3]).

The “Size fluctuation” generalization of risk model (1)\(^{[3]} \) is

\[ R_t^\rho = u + (1 + \rho)\mu A - \sum_{k=1}^{N_{A_t}} Z_k, \]  

(2)

where \( \mu = E(Z_1), \rho = \frac{c - \mu}{\mu} > 0. \)

In risk model (2), the expectation of \( A_t \) stands for the mean number of claims in interval \( (0, t] \). The numbers of total policyholders in interval \( (0, t] \) is proportional to the mean value of claims in interval \( (0, t] \) and the premium income in interval \( (0, t] \) is proportional to the total policyholders in interval \( (0, t] \).

The classical risk process perturbed by diffusion is (see [1])

\[ \bar{R}_t = u + ct + \sigma W_t - \sum_{k=1}^{N_t} Z_k, \]  

(3)

where \( \sigma > 0 \) is a constant, \( W_t \) is a standard Brownian motion which stands for the uncertainty income of an insurance company, the meaning of each signal of the rest in (3) is similar to that of the corresponding signal in (1). We now consider the “Size fluctuation” generalization of risk model (3). It is natural for us to use \( (1 + \rho)\mu A \) and \( \sum_{k=1}^{N_{A_t}} Z_k \) to describe the premium income and the total amount of claims in interval \( (0, t] \) respectively. The thing left for us now is only to choose the suitable connection between the uncertainty income and the random measure \( A_t \).

In practice, with the increments of the numbers of policyholders, or equivalently, with the increments of the value of \( A_t \), the premium income will increase, which causes the augment of the capital used to invest in. Thus the capital with uncertainty income will also be enlarged. This gives rise to increasing fluctuation of the uncertainty income, or equivalently, this yields the increments of the variance of the uncertainty income. For simplicity, we consider the situation where the variance of the uncertainty income is proportional to the total policyholders in interval \( (0, t] \), or equivalently, proportional to the mean value of \( A_t \). Therefore the “size fluctuation” generalization of risk model (3) should be

\[ \bar{R}_t = u + (1 + \rho)\mu A + \sigma \bar{W}_t - \sum_{k=1}^{N_{A_t}} Z_k, \]  

(4)