EXISTENCE OF EXPECTATION EQUILIBRIUM OF REAL ASSET ECONOMIES WITH TRANSACTION COSTS

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Abstract. To some two-period economies with countable infinite state spaces, the existence of expectation equilibrium of real asset economies with transaction costs is given. This work extends the researches of Zame in 1993.

§ 1 Introduction

Radner\(^{[1]}\) gave the existence of rational expectation equilibrium in a sequence of markets under uncertainty with finite state spaces and periods when the assumption of the continuity and convexity of preference are satisfied and the positions of all assets held are consistent and bounded. Most of the researches following focused on the role of Radner's condition consistency and boundedness. Hart\(^{[2]}\) proved its generic indispensability and the randomness of boundedness given to the positions of assets held, i.e. the boundedness isn't the natural fruit of bankruptcy. Cass\(^{[3]}\) and Werner\(^{[4]}\) found that the limits on assets holding to vouch the existence of equilibrium are unnecessary when all assets are denominated in a single commodity. More generally, \(^{[5]}\) showed that the inexistence discussed in \(^{[2]}\) is insignificant in a sense of universality.

We deal with the two-period real asset economies with countable infinite state spaces in this paper. Though we can't expect a usual equilibrium, we do expect the existence of expectation equilibrium when taking into account the transaction costs. Zame\(^{[6]}\) discussed the existence of equilibrium when working with numeraire securities, i.e. securities are denominated in a single commodity. We expands his work with the additional condition that the assets trade with corresponding fees and give the existence of expectation equilibrium of real asset market economies.

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§ 2 The Model

Considering a two-period economy, transaction occurs (at date 0) in assets and then (at date 1) in real commodities. The uncertainty of date 0 and date 1 is described by an atomic probability space $(S, \sigma)$, where $S = \{1, 2, \ldots, s, \ldots\}$ is the set of states of nature, and $\sigma$ is a probability measure on $S$. We assume that $\pi(s) > 0$, for each $s \in S$. At each state there are $l$ physical goods available for consumption. Consumption bundles are the elements of the commodity space $L_1(S, \sigma)'$; i.e. function $x : S \rightarrow \mathbb{R}^l$ for which the norm

$$\|x\| = \int |x(s)| d\sigma(s) = \int \sum_{i=1}^l |x_i(s)| d\sigma(s),$$

is finite, where $x \in L_1(S, \sigma)'$. An asset (or security) is a claim to a numeraire pattern at date 1. The return on asset $a$ in state $s$ is $a(s)$, which is an element of $\mathbb{R}^l$ and the payoff at date 1 denominated in the quantity of commodity. We generally assume that assets returns are uniformly bounded.

If there are $N$ assets $a_1, a_2, \ldots, a_N$, a portfolio is a vector $y = (y_1, y_2, \ldots, y_N) \in \mathbb{R}^N$; $y_k$ is the holding of the $k$th asset, which may be positive, negative or zero. The return on the portfolio $y = (y_1, y_2, \ldots, y_N)$ is:

$$R(y) = \sum_{k=1}^N y_k a_k \in L_1(S, \sigma)'.$$

Asset prices are the vector $a = (a_1, a_2, \ldots, a_N) \in \mathbb{R}^N$, where $a_k$ is the price of the $k$th asset. We also have the vector $C = (c_1, c_2, \ldots, c_N) \in \mathbb{R}^N_{++}$, which is the proportion of the transaction costs.

Commodity price is the function $p : S \rightarrow \mathbb{R}$, $p(s, i)$ is the price of commodity $i$ in state $s$. We shall always normalize for each state $s$, $\sum p(s, i) = 1$. (This is a free normalization, because there will be a different budget constraint in each state.)

Consumers $h \in \{1, 2, \ldots, H\}$ are defined by consumption sets $X_h$, initial endowment $e^h$, and preferences $\preceq^h$. We assume that $X_h = [L_1(S, \sigma)']_+$ for each $h$, $e^h(s) \not\in 0$ for each $s \in \{1, 2, \ldots, s, \ldots\}$ and preferences are norm continuous, convex and strictly monotone. Such preferences are representable by a continuous, quasiconcave, strictly monotonic utility function $u_h : [L_1(S, \sigma)']^+ \rightarrow [0, +\infty)$. We assume that the commodity is desirable at date 1, in the sense that, for each $x \in [L_1(S, \sigma)']_+$ there is a $c > 0$ such that $x \preceq^h c \times \chi(0, 1)$, where $\chi(0, 1)$ is the consumption pattern that consumes one unit of commodity in state 0 and nothing at date 1.

We assume that all assets trade in the exchange owned by the private, and $\lambda_h$ is the share of the exchange held by consumer $h$. We then have $\lambda_h \geq 0$ and $\sum \lambda_h = 1$.

We consider only the assets with limited liability, i.e. $a(s) \geq 0$ for each $s$. The equilibrium prices of such assets are positive.

Given securities prices $q$, commodity prices $p$ and profits of the exchange $\pi$, the bud-