ALGEBRAIC MULTI-GRID METHOD IN TWO-DIMENSION ELECTRICALLY LARGE PROBLEMS*

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Abstract In this paper the algebraic multi-grid principle is applied to the multilevel moment method, which makes the new multilevel method easier to implement and more adaptive to structure. Moreover, the error spectrum is analyzed, and the reason why conjugate gradient iteration is not a good relaxation scheme for multi-grid algorithm is explored. The numerical results show that our algebraic block Gauss Seidel multi-grid algorithm is very effective.

Key words Moment method; Algebraic multi-grid method; Block Gauss Seidel algorithm

I. Introduction

The moment method (MM)\cite{1} is well known as a powerful means for analyzing electromagnetic problems. The application of this method has been limited to the lower frequency problem because of the high computational cost of solving matrix equation which is of the order $O(N^3)$ if Gaussian elimination is used or $O(N^2)$ per iteration if the conjugate gradient (CG) method is used. During the last decade, many researchers have attempted to reduce the computational complexity of the traditional MM algorithm by reducing the computational labor of the pertinent matrix-vector multiplication\cite{2,3}.

Multi-grid method takes a different approach to electrically large problems. The principle of multi-grid algorithm was first presented by Brandt\cite{4} for solving boundary-value problems and later developed by a lot of researchers\cite{5-7} for various applications. Among these applications, Kalbasi proposed multilevel moment method (MLMM)\cite{5} aimed at solving an integral equation for scattering problems. In his work, the basis functions chosen for both fine and coarse grids of discretization must be of same type.

In this paper, effort is also devoted to solving an integral equation for an electromagnetic problem, but the basic idea is based on the principle of algebraic multi-grid algorithm\cite{8}. In so doing, the transfer from one level to another is fulfilled by a general basis transformation, the basis functions can be chosen more generally, and may or may not be of the same type. Moreover, the errors in multi-grid cycles are minimized. This paper is organized as follows. Section II outlines the principle of algebraic multi-grid algorithm. Section III describes the computational cost, explores the reasons for poor performance of conjugate gradient multi-grid method (CGMG) and proposes the Block Gauss Seidel multi-grid algorithm. Section IV presents the numerical results and Section V gives the final conclusion.

II. Algebraic Multi-Grid Method

In the numerical analysis of an electromagnetic problem, an operator equation is always

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discretized into a matrix equation

$$Ax = b$$

(1)

where $A$ is the impedance matrix, $x$ the current distribution, and $b$ the incident field.

Generally, there is no problem for an iteration algorithm to suppress the high-frequency error components in the solution, but the low-frequency error components are difficult to suppress. It is these components which cause the stagnation or eventual divergence of the process. By using a number of discretization grids, the low-frequency error components are suppressed on coarser grids while the high-frequency components are suppressed on finer grid. It is because of this reason that the multi-grid technique works quite well. We start the multi-grid (MG) process on the finest grid by finding iteration solutions $x^h$ of Eq.(1) until the convergence becomes slow, then switch to the coarser grid level. If we denote the solution error and the residual error by $e^h$ and $r^h$ respectively, then

$$A^h e^h = r^h$$

(2)

where $A^h$ denotes the impedance matrix on the fine grid. The residual error contains the low-frequency modes that are not readily accessible to the iteration process on the fine grid.

To continue the solution process on the coarser grid, let us assume that there exists a pair of operators, restriction operator $R$ and interpolation operator $I$ that project the residual error from the finer grid to the coarser grid, and vice versa. Using the restriction operator, we project $r^h$ to the coarser grid and denote it by $r^H$, yielding $r^H = Rr^h$. The residual equation on the coarser grid becomes

$$A^H e^H = r^H$$

(3)

where $A^H$ denotes the impedance matrix on the coarse grid.

The $e^H$ can be solved exactly if the number of the unknowns is small enough, or by iteration if the number of unknowns is relatively large. Once $e^H$ is obtained, it is projected to the fine grid via the interpolation operator:

$$\hat{e}^h = I e^h$$

(4)

A new estimation of the solution is then available by adding $\hat{e}^h$ to the previous estimated solution at fine level:

$$x = x^h + \hat{e}^h$$

(5)

In the MLMM, since the impedance matrices on coarser grids are obtained by using the same basis function, the restriction operator must preserve the same shape of the basis function. The restriction operator for the pulse basis function is of the following form:

$$R = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}_{N \times N}$$

(6)

The interpolation operator $I$ is related to $R$ through transposition.