TRANSPORT ANALYSIS OF
TRANSMISSION LINE CIRCUITS BASED ON
THE SEMIDISCRETIZATION OF TELEGRAPH EQUATIONS

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Abstract A new transient analysis method for the transmission line circuits is presented in
this paper. Based on the semidiscretization of the telegraph equations, a discretized time domain
companion models for the transmission lines which can be conveniently implemented in a general
circuit simulator such as SPICE is derived. The computation required for the model is linear with
time, equivalent to the recursive convolution-based method. The formulations for both single and
coupled lossy transmission lines are given. Numerical experiments are carried out to demonstrate
the validity of the method.

Key words VLSI interconnects; Transmission line analysis; Circuit simulation

I. Introduction

As the operating speed and layout density is ever increased, the transmission-line effects
of the interconnects in high speed VLSI circuits and systems are becoming more and more
significant. Accurately simulating the behavior of the interconnects with terminating loads
has gradually been an indispensable step in the design of these circuits. Numerous methods
to analyze the transient response of the transmission line circuits have therefore emerged in
the last decade\(^1\,2\).\n
Most of the analysis methods now existing can be essentially divided into two classes.
The first class of methods attempts to find a discrete equivalent time domain model for the
lines and then implement it into a general circuit simulator such as SPICE. This is always
carried out by discretizing the convolution expression of the terminal voltages and currents
relations of the transmission lines\(^1\). Since the quadratic computation complexity of the
convolution, the rational approximation, especially the Padé approximation, is then used
for the transmission line characteristics and this leads to the well-known AWE (Asymptotic
Waveform Evaluation), or the moment-matching technique, by which the convolution can be
computed recursively\(^3\). Of course, after getting a rational approximation of the transmission
lines, the discrete model can also be derived by transforming it into an ordinary differential
equations\(^4\). The numerical instability in moment-matching has been a disturbing problem
in the application of this method\(^6\). However, the Krylov subspace techniques such as
PVL (Padé Via Lanczos) introduced in recent years improves this significantly\(^6\), and some
algorithms that can preserve passivity in the approximation and so guarantee the stability
of the reduced order model finally gotten have been developed\(^7\,8\). An alternative in this
class of methods is the characteristic method\(^9\,10\), which obtains the discrete model by
the discretization of the ordinary differential equations along the characteristic line of the
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The second class of methods try to approximately synthesize the characteristics of the transmission lines in the frequency domain with lump RLC elements and ideal delay lines and then accomplish the analysis using ordinary circuit simulators. This is in fact the principal approach for the transmission line analysis in the early stage. More efficient approximate techniques such as Pade approximation are adopted in the recent years\cite{11,12}. Except these two class of methods, there are also some other methods such as the waveform relaxation\cite{11}, the primary discretization method which discretizes both the time and space differentials in the telegraph equations\cite{1}. However, they are more complex, tedious, inefficient and inconvenient to use.

A new analysis method is proposed in this paper. It also tries to find a discrete time domain model and so is similar to the first class of methods above. However, unlike the usual way starting from the frequency domain description of lines and then making rational approximation, our method obtain the discrete model directly in the time domain based on the semidiscretization of the telegraph equations. Discretizing the time derivatives in the telegraph equations using appropriate numerical formula while remain the space derivatives unchanged, we get an ordinary differential equations which can be solved analytically. Its solution yields a discrete time domain companion model which can be implemented conveniently in a general circuit simulator such as SPICE. The complexity of the computations required for the model is linear, equivalent to the convolution-based model using recursive technique. Besides, for the lossless line, the limiting case of the time step \( h \to 0 \) is the same as the Branin's classic characteristic model.

The paper is organized as follows: Section II describes the basic idea of the method and derives the discrete model for single conductor line. Section III extends it to the multiconductor coupled lines case. Some problems when implementing the model in a circuit simulator are discussed in Section IV. Experimental results are given in Section V. Finally, Section VI draws the conclusion of the paper.

II. Derivation of the Model for Single Line

Consider the telegraph equations for a lossy, uniform RLCG transmission line,

\[
\frac{\partial v(x,t)}{\partial x} = -Ri(x,t) - L \frac{\partial i(x,t)}{\partial t}, \quad \frac{\partial i(x,t)}{\partial x} = -Gv(x,t) - C \frac{\partial v(x,t)}{\partial t}
\]  

(1)

in which \( x \) varies from 0 to \( l \), \( l \) is the length of line. Generally, it is impossible to deal with Eq.(1) analytically in the time domain, and almost all of the previous methods rely on the Laplace transformed solution in the frequency domain. However, what really needed in modern numerical circuit simulation for a circuit component is only a discretized time domain model, i.e., the companion model, not an analytical solution. This observation motivates us to find a discrete numerical model directly from Eq.(1). As the usual way to construct the discrete model for lump elements, we discretize the derivatives with respect