APPLICATION OF FM^mlette TRANSFORM TO SIGNAL SEPARATION*

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Abstract Since the multicomponent signal with nonlinear time-frequency structures occupies a wide frequency band, and the spectral contents may alias, it is therefore difficult to separate the signal components and to separate the signal from background noise. In this paper, a new signal separation method using FM^mlette transform is proposed by taking the advantage that the atoms of FM^mlette transform can match both the linear and nonlinear time-varying structures. Theoretical predictions and numerical experiments show the feasibility of the methodology advocated.

Key words FM^mlette transform; Parametric atomic decomposition; Matching pursuit; Signal separation

I. Introduction

Two main pillars of signal processing technology are filter design and spectral analysis. The commonly used filtering techniques based on spectrum windowing, wavelet packets, chirplets, and masked time-frequency planes are not satisfactory for signals that include linear and nonlinear time-varying components. In this paper, we propose the FM^mlette transform [1] to perform the signal separation. Since some of the existing transforms like Fourier transform, short-time Fourier transform (including Gabor transform), wavelet transform, and chirplet transform may each be found to be a special case of the FM^mlette transform with specific parameters, and many interesting physical and artificial processes yield linear and nonlinear time-varying phenomena, the FM^mlette transform is therefore more flexible for denoising and interference cancellation.

II. Outline of FM^mlette transform

The choice of atoms plays the most important role in linear signal representations. In order to represent a signal concisely and efficiently, the atoms should be adapted to the signal’s local structures. While several transforms such as Fourier transform, short-time Fourier transform (or Gabor transform), and wavelet transform have already been proposed to characterize the signal’s structures, none has been able to adequately handle the chirp-like signals; the newly proposed chirplet transform is only suitable for analyzing the signals whose frequency contents vary linearly with time. A common shortcoming with all of these transforms is however their failure to capture the true time-varying nature of the signals with nonlinear time-frequency structures.

Contemplating the general drawback of the atoms whose spectral components change only linearly over time, we consider to modify the chirplet atom by the simple expedient of appending just one extra parameter, an exponent, to the frequency modulation term in the chirplet atom[2], to accommodate both the linear and nonlinear time-frequency behaviors in a class of man-made and natural signals such as radar impulses, whistlers, dispersed surface waves in seismic signatures, EEG signals, bird songs, and bat sonar signals. Let m designate

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the exponent (hereafter "FM exponent"). Adding $m$ to the chirplet leads to a dilated and translated windowed exponential frequency modulated atom

$$q_{t_c,f_c,\log \sigma,r,m}(t) = \frac{1}{\sqrt{\sigma}} g\left(\frac{t-t_c}{\sigma}\right) \exp\left\{j2\pi \left[1 + r \left(\frac{t-t_c}{\sigma}\right)\right]^m f_c \left(\frac{t-t_c}{\sigma}\right)\right\}$$

(1)

which for convenience we shall call "FM$m$let" in accordance with the common convention in nomenclature in signal processing community. From the perspective of etymology, the anagram "FM$m$let" is born by combining the "FM" (exponential frequency modulation) with the diminutive "let" (smallness, which here refers to windowing manipulation), thus its concept takes shape.

Akin to the mother wavelet in wavelet theory, the window function $g(t)$ may be viewed as the primitive that generates a dictionary of FM$m$let atoms, we will, therefore, refer to this primitive as the "mother FM$m$let". Clearly, such an FM$m$let atom is dictated by, besides the shape of $g(t)$, the following five free parameters that each have an intuitively satisfying significance: time-center $t_c$, frequency-center $f_c$, log-duration $\log \sigma$, chirprate $r$, and FM exponent $m$.

Eq.(1) places in evidence the fact that, (1) if $m = 1$, Eq.(1) will reduce to chirplet; (2) if $m = 0$, Eq.(1) will reduce to wavelet; (3) if $m = 0$ and $\sigma = 1$, Eq.(1) will reduce to the atom of short-time Fourier transform (including that of Gabor transform); (4) if $m = 0$, $\sigma = 1$, $t_c = 0$, and $g(t) \equiv 1$ (viz., the window is abandoned), then the FM$m$let will reduce to a pure sinusoidal wave which is just the elementary function of Fourier transform.

Once the FM$m$let atoms are used as elementary functions, the FM$m$let transform of any square-integrable signal $s(t) \in L_2(\mathbb{R})$ may be readily defined as

$$FM^mT(t_c,f_c,\log \sigma,r,m) = \langle s(t), q_{t_c,f_c,\log \sigma,r,m}(t) \rangle$$

$$= \frac{1}{\sqrt{\sigma}} \int_{-\infty}^{\infty} s(\nu) g^*(\nu) \frac{(t-\nu)}{\sigma} \exp\left\{-j2\pi \left[1 + r \left(\frac{t-\nu}{\sigma}\right)\right]^m f_c \left(\frac{t-\nu}{\sigma}\right)\right\} d\nu$$

(2)

where \(\langle , \rangle\) denotes the Dirac inner product, which gauges the level of similarities between the atoms and the signal under analysis. The superscript \(^*\) denotes the complex conjugate operation, and \((t_c,f_c,\log \sigma,r,m)\) is the parameter list.

III. Application of FM$m$let transform to signal separation

Let $s \in L^2(\mathbb{R})$. Our objective is to calculate a linear expansion of $s$ over a set of vectors selected from $D$, which best matches the components of $s$. This kind of expansion is realized by orthogonal projections of $s$ on vectors of $D$. Let $q_{\gamma_0} \in D$, then the signal $s$ can be decomposed into

$$s = \langle s, q_{\gamma_0} \rangle q_{\gamma_0} + R_s$$

(4)

where \(\langle s, q_{\gamma_0} \rangle q_{\gamma_0}\) is the projection of $s$ in the direction of $q_{\gamma_0}$, $R_s$ is its corresponding residual signal. Obviously, $q_{\gamma_0}$ is orthogonal to $R_s$, \(\|q_{\gamma_0}\| = 1\), so

$$\|s\|^2 = \|\langle s, q_{\gamma_0} \rangle q_{\gamma_0} \|^2 + \|R_s\|^2.$$  

(5)

The matching pursuit based FM$m$let transform is an iterative projection algorithm that subdecomposes the residual signal $R_s$ by projecting it on a vector of $D$ that matches $R_s$ almost best, as it was done for $s$ in Eq.(4). After each iteration, an FM$m$let atom that best matched the dominating component of residual signal is selected. The decomposition process will stop when certain criterion such as the ratio between the energy of remainder