SOLVING THE PROPAGATION OF ELECTROMAGNETIC WAVE IN A SIMPLE TWO-DIMENSIONAL INHOMOGENEOUS MEDIUM BASED ON SYMPLECTIC GEOMETRICAL THEORY

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Abstract A new symplectic geometrical high-frequency approximation method for solving the propagation of electromagnetic wave in the two-dimensional inhomogeneous medium is used in this paper. The propagating caustic problem of electromagnetic wave is translated into non-caustic problem by the coordinate transform on the symplectic space. The high-frequency approximation solution that includes the caustic region is obtained with the method combining with the geometrical optics. The drawback that the solution in the caustic region can not be obtained with geometrical optics is overcome by this method. The results coincide well with that of finite element method.

Key words Symplectic geometrical theory; High-frequency approximation; Caustics; Two-dimensional inhomogeneous medium

I. Introduction

Few problems involving wave propagation in a spatially inhomogeneous medium lead to exact solutions. Consequently, various kinds of approximate methods have been presented to obtain solutions under different conditions. Geometrical Optics (GO) approximation is one of the most successful methods for evaluating high frequency field in the inhomogeneous medium. However, it predicts an infinite field in the vicinity of a caustic region. In mathematical point of view, there is a singularity in the caustic region in physical space. But the singularity is not a real one. In fact, the solutions of electromagnetic wave equation are not singular. It is because the primarily simple expression of GO approximation is not suitable for the caustic region that the solutions are singular. A systematic procedure which remedies these defects has been proposed by Maslov\(^1\). This method put the GO field into the phase space \( M \) which has the double dimensions of the physical space, \( M = X \times K \), where \( X \) is the physical space, \( K \) is the wave space which has the same dimensions with the physical space. The rays in the phase space have no singularity. At the singular point of the ray expression in the physical space, we can project the rays in the phase space to the hybrid space which consists of part vectors in the physical space and part vectors in the phase space. We can use the high-frequency approximation in the hybrid space, then transform the solution to the physical space by Fourier transform with parameter. It has been proved mathematically that we always can find such a hybrid space which has no singularity in the singular point of the physical space. For Maslov's method is a high-frequency approximation, we call it symplectic geometrical high-frequency approximation method. This method has been suc-

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cessfully applied to calculate the filed in one-dimensional inhomogeneous medium\cite{2}. In this paper, the method for calculating wave field of the caustic region in the two-dimensional inhomogeneous medium is given through an example of a simple two-dimensional inhomogeneous medium. Finally the results are compared with those obtained by finite elements method. The solutions fit very well.

II. The Electromagnetic Wave Equation in the Two-Dimensional Inhomogeneous Medium

In this paper we consider that a plane wave is incident into an isotropic two-dimensional inhomogeneous medium. The magnetic permeability of the medium is a constant $\mu_0$. The dielectric constant of the medium varies in two dimensions and is given in the form:

$$\varepsilon(x, z)\mu_0 = 1 - \alpha x^2 - \beta z^2$$  \hspace{1cm} (1)

So the electromagnetic equations in the two-dimensional medium can be written as

$$\nabla^2 u + \omega^2 (1 - \alpha x^2 - \beta z^2) u = 0$$  \hspace{1cm} (2)

III. Solving the Wave Equation in the Two-Dimensional Medium Using Symplectic Geometrical High Frequency Approximation Method

The incident plane wave can be written in the form:

$$u(x, z, \omega) = e^{i\omega(\xi_0 - \zeta_0 \omega)}$$  \hspace{1cm} (3)

where $\xi_0 = \sin \theta$, $\zeta_0 = \cos \theta$, $\theta$ is the incident angle.

Use GO approximation to solve the high frequency field of the wave Eq.(2). Assume the solution is

$$u(x, z, \omega) = a(x, z, \omega)e^{i\omega\phi(x, z)}$$  \hspace{1cm} (4)

then eikonal equation

$$\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial z}\right)^2 = 1 - \alpha x^2 - \beta z^2$$  \hspace{1cm} (5)

is obtained.

Introduce two new variables $\xi = \frac{\partial \phi}{\partial x}$, $\zeta = \frac{\partial \phi}{\partial z}$, and consider the four-dimensional phase $N(x, z, \xi, \zeta)$. The eikonal equation Eq.(5) is transferred to an algebraic equation.

$$\xi^2 + \zeta^2 - (1 - \alpha x^2 - \beta z^2) = 0$$  \hspace{1cm} (6)

Take the Hamilton function as

$$H(x, z, \xi, \zeta) = \frac{1}{2}[\xi^2 + \zeta^2 - (1 - \alpha x^2 - \beta z^2)]$$  \hspace{1cm} (7)

then the problem of tracing the rays in the physical space $X(x, z)$ is replaced by the first order ordinary differential equations:

$$\begin{align*}
\frac{dx}{dt} &= \frac{\partial H}{\partial \xi} = \xi \\
\frac{dz}{dt} &= \frac{\partial H}{\partial \zeta} = \zeta \\
\frac{d\xi}{dt} &= -\frac{\partial H}{\partial x} = -\alpha x \\
\frac{d\zeta}{dt} &= -\frac{\partial H}{\partial z} = -\beta z
\end{align*}$$  \hspace{1cm} (8)