ANALYSIS AND SIMULATION OF QSP BEAMFORMER

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Abstract The real Direction Of Arrival (DOA) varies with time in mobile communication system. In such situation, the performance of conventional beamformers will be degraded obviously. Quantum Signal Processing (QSP) beamformer is insensitive to DOA errors, thus it can achieve stable output performance in such circumstance. This letter verified the effectiveness and feasibility of the QSP beamformer by simulation results.

Key words Beamforming; DOA error; Quantum Signal Processing (QSP); Impact factor

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I. Introduction

Beamforming[1,2], which is the key technology of the smart antenna, is getting more and more attention as the smart antenna is adopted in the 3rd generation mobile communications. Conventional beamforming, such as Capon beamformer, is widely used since it only needs Direction Of Arrival (DOA) of the interested signal and can be easily implemented. But in mobile environment the DOAs vary with time because of the mobility of the users, which results in instable output performance of the conventional beamformer[3–5]. Adaptive algorithm, as a choice in such environment, is restricted in the application by the convergence speed as well. Presented in Ref.[6], the Quantum Signal Processing(QSP) beamformer is based on quantum signal processing framework, and is discussed as a new method of solving this problem. It can provide not only good but also stable output performance in such environment because of its insensibility for random perturbations on the DOAs of signals.

II. Signal Model[1]

Assume that there are $J$ narrow-band stationary signals in far field $\{i_j(t), j = 1,2,\cdots,J\}$, and the DOAs of the signals are $\{\theta_j, j = 1,2,\cdots,J\}$ respectively. The receiver antenna array is composed of $M$ elements. Suppose that the Additive White Gaussian Noise (AWGN) at each array element is $\{n_m(t), m = 1,2,\cdots,M\}$ with the same variance $\sigma^2$, and the vector form of the received signals is

$$x(t) = As(t) + n(t) = \sum_{j=1}^{J} a(\theta_j)i_j(t) + n(t) \quad (1)$$

where $a(\theta_j) = [a_1(\theta_j) \cdots a_M(\theta_j)]^\top$ is the steering vector corresponding to the signal from DOA $\theta_j$, $x(t) = [x_1(t) \cdots x_M(t)]^\top$, $A = [a(\theta_1) \cdots a(\theta_J)]$, $s(t) = [i_1(t) \cdots i_J(t)]^\top$, and $n(t) = [n_1(t) \cdots n_M(t)]^\top$.

According to the principle of beamforming, after adjusting the weight vector $w$ of the array, we can get the output signal of the beamformer, which is denoted as: $y(t) = w^\top x(t)$. With this method, the jamming can be reduced and the expected signals can be better separated.

In the mobile environment, because of the mobility of the user, random errors exist on a fixed estimation of some DOA, and make the real steering vector different with the ideal steering vector. We denote the errors as a Gaussian random vector $\theta_\varepsilon$, which is an addition for the ideal DOA. The mean of $\theta_\varepsilon$ is 0 and the standard deviation is $\sigma_\varepsilon$. So the real steering vector is not $a(\theta_j)$, but $a(\theta_j + \theta_\varepsilon)$, and the real data at the receiver can be written as

$$x(t) = A's(t) + n(t) = \sum_{j=1}^{J} a(\theta_j + \theta_\varepsilon)i_j(t) + n(t) \quad (2)$$

The estimation of the covariance of received data based on Eq.(2) is not accurate and it will lead
to the loss of the output performance.

### III. Performance Analysis of the QSP Beamformer

For conventional Capon beamformer, the weight vector $\mathbf{w}$ is denoted as $\mathbf{w} = \mu \mathbf{R}^{-1} \mathbf{a}(\theta_j)$, where $\mathbf{R}$ is the covariance of the received data, and the output of the beamformer is good when the DOAs of the signals are known accurately. But the DOA errors exist in mobile environment, the conventional Capon beamformer can not obtain the optimal value of $\mathbf{w}$ timely, and then the output performance of the beamformer will be degraded obviously.

QSP beamformer[6] is a new kind of beamformer based on quantum signal processing framework[7]. Its principle can be expressed as follows:

Assume that there are $J$ steering vectors $\{\mathbf{a}(\theta_j), 1 \leq j \leq J\}$ in Hilbert space $\mathcal{H}$. When a group of vectors $\{\mathbf{a}(\theta_j), 1 \leq j \leq J\}$ are given, we can construct a group of vectors $\{\mathbf{w}(\theta_j), 1 \leq j \leq J\}$ whose element is orthogonal to each other, which approach to the given group of vectors $\{\mathbf{a}(\theta_j), 1 \leq j \leq J\}$ under least-square constraint as close as possible. Furthermore, we introduce different impact factors $\{q_j, 1 \leq j \leq J\}$ for different signals, which satisfy $q_1 + q_2 + \cdots + q_J = 1$, then the vectors $\{\mathbf{w}(\theta_j), 1 \leq j \leq J\}$ should make the following equation minimum

$$
\varepsilon_{LS} = \sum_{j=1}^{J} q_j \left( \mathbf{a}(\theta_j) - \mathbf{w}(\theta_j), \mathbf{a}(\theta_j) - \mathbf{w}(\theta_j) \right)
$$

s.t. $\{\mathbf{w}(\theta_j), \mathbf{w}(\theta_j)\} = c^2 \delta_{jj}$

$$
q_1 + q_2 + \cdots + q_J = 1
$$

where $c$ is a constant value greater than zero, $\delta_{jj}$ indicates that the vectors should be orthogonal to each other and the constant $q_j$ denotes the impact factor of the $j$-th signal. After calculation we can get

$$
\mathbf{w} = \mathbf{a}(\theta_j) \mathbf{Q} \left( \mathbf{Q}^* \mathbf{A} \mathbf{Q} \right)^{-1}, \{1 < j < J\}
$$

where $\mathbf{A}$ is the matrix composed of all steering vectors, $(\cdot)^j$ denotes the Moore-Penrose pseudo-inversion and $\mathbf{Q}$ is a diagonal matrix $\mathbf{Q} = \text{diag}\{q_1, q_2, \ldots, q_J\}$.

From Eq.(4), we know that the stability of $\mathbf{w}$ in QSP beamformer is better than the one in the Capon beamformer, because the $(\cdot)^{-1/2}$ is more stable than $(\cdot)^{-1}$ when $(\cdot)$ changes slightly.

In order to validate the stable performance of QSP beamformer, we present the simulation as follows.

**Simulation environment** We utilize a four-element Uniform Linear Array (ULA) with the elements separated by half-wavelength interval and assume that the expected signal comes from the direction of $0^\circ$, and an interference signal comes from the direction of $15^\circ$, and Signal to Interference Ratio (SIR) is 0dB. The impact factor of the expected signal is $q_1 = 0.1$ and $q_2 = 0.9$. The result is shown in Fig.1.

**Fig.1** The output SINR of the beamformers

In Fig.1, the output Signal to Interference and Noise Ratio (SINR) curves of the two beamformers are drawn. When DOAs of all signals are stable and accurately known, Capon beamformer presents its excellent performance as curve Capon shows in Fig.1, and there is a performance gap between the curve QSP which represents the output performance of QSP beamformer and curve Capon. Along with the SNR goes higher, the gap goes wider.

The curve Capon 01 and the curve Capon 03 represent the outputs of Capon beamformer in the situation where the DOA errors exist with $\sigma_\theta = 0.1$ and 0.3 separately. And the curve QSP 01 and curve QSP 03 represent the outputs of QSP beamformer with the same environment. From Fig.1, it is obviously found that the performance of the Capon beamformer is rapidly degraded as the variance of random errors gets larger, especially when the SNR is high. Meanwhile, the performance of the QSP beamformer is only degraded partly