A STUDY ON EM SCATTERING OF A TWO-DIMENSIONAL INHOMOGENEOUS STRUCTURE BURIED IN STRATIFIED MEDIA*

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Abstract  The TM-polarized electromagnetic scattering problem of a two-dimensional inhomogeneous structure buried in lossy stratified media is presented. Analysis work consists of two parts mainly, derivation of the Green’s function of a filament buried in lossy stratified media and constitution of the electric field integral equation of the equivalent current caused by the differences between the inhomogeneities and the stratified media. Based on these works, illustrative numerical results are given to model inhomogeneous underground tubes in lossy stratified media, and to describe the scattering field affected by different factors such as permittivity distribution, dimension, and buried depth of the inhomogeneities and so on.

Key words  EM Scattering; Stratified media; 2-D inhomogeneity; Buried object

I. Induction

The electromagnetic scattering problem of inhomogeneities in lossy stratified media is significant. It can be used for modeling purposes in many fields such as nondestructive detection in municipal engineering and industrial process, remote sensing of subsurface materials, CAD of microwave device and so on. The past works are mostly concerned with the problem of an object buried in half-space. In order to model practical problems more precisely, the paper extends to the TM-polarized EM scattering of an inhomogeneous structure buried in arbitrary stratified media. The paper is organized as follows: Section II gives the 2-D Green’s function in stratified media. Section III constitutes the electric field integral equation of the equivalent current related to inhomogeneous structure. Some numerical results and analyses are given in Section IV.

II. 2-D Green’s Function in Stratified Media

As shown in Fig.1, N parallel interfaces $z_1, z_2, \ldots, z_N$ ($z_1 = 0$) separate the space into $N + 1$ stratified regions $\Delta z_0, \Delta z_1, \Delta z_2, \ldots, \Delta z_N$. We may assume that the region $\Delta z_0$ is free space with wavenumber $k_0$, and the rest regions are different lossy media with $k(z) = k_n, z \in \Delta z_n$. In the model $k_n$ may be complex for a lossy layer. When a unity y-direction filament $\delta(x - x')\delta(z - z')\hat{y}$ is located in $l$-th layer, the electric field $E_y(x, z)$ in all regions will meet the following wave equation:

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Taking Fourier transform to $x$, we have 1-D wave equation,
\[(\frac{\partial^2}{\partial z^2} + k^2(z) - \alpha^2)\tilde{E}_y = -i\omega\mu_0\frac{e^{-i\alpha z'}}{\sqrt{2\pi}}\delta(z - z')\]

It seems that 1-D electric field $\tilde{E}_y(z')$ is supported by a surface current $\frac{e^{-i\alpha z'}}{\sqrt{2\pi}}$ in the interface $z = z'$. From the viewpoint of the circuit theory, this surface current is supplied to two parallel loads, the downward input impedance $Z_{\text{down}}(z')$ and upward input impedance $Z_{\text{up}}(z')$ in $z = z'$, they are defined as

\[Z_{\text{down}}(z') = \frac{\tilde{E}_y(z'^+)}{-\tilde{H}_z(z'^+)}, \quad Z_{\text{up}}(z') = \frac{\tilde{E}_y(z'^-)}{\tilde{H}_z(z'^-)}\]

where $z'^-$ and $z'^+$ represent the up and down side of the interface $z = z'$ respectively. By using the propagation matrix\[^{[1]}\], $Z_{\text{down}}(z')$ and $Z_{\text{up}}(z')$ can be expressed in terms of $W_N$ and $W_0$ which are the wave impedance in region $N$ and region 0 respectively, i.e.

\[Z_{\text{down}}(z') = \frac{A_{z'^+,z_N} W_N + B_{z'^+,z_N}}{C_{z'^+,z_N} W_N + D_{z'^+,z_N}}, \quad Z_{\text{up}}(z') = \frac{A_{z'^-,z_1} W_0 + B_{z'^-,z_1}}{C_{z'^-,z_1} W_0 + D_{z'^-,z_1}}\]

where the coefficients $A, B, C, D$ are the elements of the propagation matrix respectively, the subscript denotes two referred interfaces. In detail there are

\[\begin{pmatrix} \tilde{E}_y(z'^+) \\ -\tilde{H}_z(z'^+) \end{pmatrix} = \begin{pmatrix} A_{z'^+,z_N} & B_{z'^+,z_N} \\ C_{z'^+,z_N} & D_{z'^+,z_N} \end{pmatrix} \begin{pmatrix} \tilde{E}_y(z_N) \\ -\tilde{H}_z(z_N) \end{pmatrix}\]

\[\begin{pmatrix} \tilde{E}_y(z'^-) \\ \tilde{H}_z(z'^-) \end{pmatrix} = \begin{pmatrix} A_{z'^-,z_1} & B_{z'^-,z_1} \\ C_{z'^-,z_1} & D_{z'^-,z_1} \end{pmatrix} \begin{pmatrix} \tilde{E}_y(z_1) \\ \tilde{H}_z(z_1) \end{pmatrix}\]

From the boundary conditions of electric and magnetic field, we can determine the tangential field components on the two sides $z'^\pm$:

\[\tilde{H}_z(z'^+) = \frac{Z_{\text{up}}(z') e^{-i\alpha z'}}{Z_{\text{up}}(z') + Z_{\text{down}}(z')}\]