STABILITY OF BIDIRECTIONAL ASSOCIATIVE MEMORY NEURAL NETWORKS WITH DELAYS

Liao Xiaoxin
(Dept. of Auto. Control. Huazhong Univ. of Science & Technology, Wuhan 430074)

Liao Yang
(Dept. of Computer Science, Nanjing University, Nanjing 210093)

Liao Yu
(Wuhan Soundy Science & Commerce Company, Wuhan 430070)

Abstract In this paper the globally asymptotic stability of more general two-layer nonlinear feedback associative memory neural networks with time delays is examined. The sufficient conditions of existence, uniqueness and globally asymptotic stability of the equilibrium position are given. Finally, two interesting examples to illustrate the theory are given.

Key words Neural networks; Associative memories; Stability

I. Introduction

It is well known that the stability and encoding properties of two-layer nonlinear feedback neural networks without time delays were investigated by Kosko[1,2]. But in many networks, time delays cannot be avoided. Marcus and Westerval[3] proposed in a similar way as Hopfield a model for network with delays.

In this paper we consider more general bidirectional associative memory neural networks with time delays as follows:

\[
\frac{dx_i}{dt} = -\alpha_i x_i + \sum_{j=1}^{m} a_{ij} g_j(y_j(t - \tau_{ij})) + I_i, \quad i = 1, \cdots, n
\]

\[
\frac{dy_j}{dt} = -\beta_j y_j + \sum_{k=1}^{n} b_{jk} h_k(x_k(t - \tau_{jk})) + J_j, \quad j = 1, \cdots, m
\]

where \(\alpha_i, \beta_j\) are some positive constants; \(a_{ij}, b_{jk}\) are weight coefficients, we admit \(a_{ij} \neq b_{ij}\); \(\tau_{ij} > 0, \sigma_{jk} > 0\) are time delays; the constant's inputs \(I_i\) and \(J_j\) can be interpreted as the sustained environmental stimuli or as the stable reverberation from an adjoining neural network; \(k = 1, 2, \cdots, n, j = 1, 2, \cdots, m\).

We assume that output functions \(g_j, h_k\) satisfy:

\[
0 < \frac{dg_j(\xi)}{d\xi} \leq 1, \quad 0 < \frac{dh_k(\xi)}{d\xi} \leq 1, \quad j = 1, 2, \cdots, m, \quad k = 1, 2, \cdots, n
\]
II. The Existence, Uniqueness and Stability of Equilibrium Position

In this section, we firstly investigate the existence, uniqueness and globally asymptotic stability for the equilibrium points.

**Theorem 1** Assume that the matrix: 
\[ \Omega = \begin{pmatrix} \alpha & -|A| \\ -|B| & \beta \end{pmatrix} \] 
\( (n+m)(n+m) \) is an M matrix\(^4\). Then the equilibrium position \( x = x^*, y = y^* \) of Eqs.(1) and (2) is existent uniquely and it is globally asymptotically stable.

Where \( \alpha = \text{diag}(\alpha_1, \cdots, \alpha_n), \beta = \text{diag}(\beta_1, \cdots, \beta_m), |A| = (|a_{ij}|)_{n \times m}, |B| = (|b_{ij}|)_{m \times n} \).

**Proof** We firstly prove the existence and uniqueness of the equilibrium position of Eqs.(1) and (2).

Let us consider the equations:
\[ x_i = \sum_{j=1}^{m} \frac{a_{ij}}{\alpha_i} g_j(y_j) + I_i, \quad i = 1, 2, \cdots, n \]  
\[ y_j = \sum_{k=1}^{n} \frac{b_{jk}}{\beta_j} k(x_k) + I_j, \quad i = 1, 2, \cdots, m \]

By the property of M matrix\(^5\), we know that \( \Omega \) to be an M matrix implies \( \rho(H) < 1 \), where \( \rho(H) \) is the spectrum radium of matrix \( H \) and
\[ H = \begin{pmatrix} 2 & 0 \\ 0 & \beta \end{pmatrix}^{-1} \begin{pmatrix} 0 & |A| \\ |B| & 0 \end{pmatrix} \]

According to Theorem 1 in Ref.[4], \( \rho(H) < 1 \) implies the fixed point \( x = x^*, y = y^* \) to be existent uniquely, which satisfy Eqs.(3) and (4), i. e.,
\[ \alpha_i x_i^* = \sum_{j=1}^{m} a_{ij} g_j(y_j^*) + I_i \]  
\[ \beta_j y_j^* = \sum_{k=1}^{n} b_{jk} k(x_k^*) + I_j \]

Therefore \( x = x^*, y = y^* \) is unique equilibrium position of Eqs.(1) and (2).

Now, we prove \( x = x^*, y = y^* \) to be globally asymptotically stable. According to the property of M matrix\(^3\), the condition of theorem implies that there exist \( n + m \) positive