EFFICIENT DESIGN METHOD OF COSINE-MODULATED QMF BANKS SATISFYING PR PROPERTY*

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Abstract In this paper the design problem of perfect-reconstruction cosine-modulated QMF banks has been formulated as a quadratic-constrained least-squares (QCLS) minimization problem in which all constrained matrices of the QCLS optimization problem are symmetric and positive definite. A cost function which is a convex function of desired prototype filter coefficients is constructed so that this kind of QCLS optimization problems can be efficiently solved. So a global minimizer of this problem can be easily obtained. Results of two design examples are presented to support the derivations and analyses.

Key words Quadratic-constrained least-squares (QCLS); Cosine-modulated QMF banks; Global optimization; Penalty function searching (PS) approach

I. Introduction

Recently, the perfect reconstruction (PR) theory of the multirate filter bank systems has been established[1-3]. Among them, the PR cosine-modulated quadrature mirror filter (PR CM-QMF) bank has emerged as an attractive choice of filter bank with respect to implementation cost and design saving. The impulse responses of the analysis and synthesis filters are cosine-modulated versions of a prototype filter. It is shown in Ref.[3] that the 2M polyphase components of the prototype filter can be grouped into M power-complementary pairs, where each pair is implemented as a two-channel lossless lattice filter bank. The lattice coefficients are optimized to minimize the stopband attenuation of the prototype filter, but this is a highly nonlinear optimization problem with respect to lattice coefficients. Consequently, it is difficult to obtain the PR CM-QMF bank with high stopband attenuation. Very recently, Refs.[4,5] formulated the design problem as a quadratic-constrained least-squares (QCLS) problem, and obtained the PR CM-QMF bank with high stopband attenuation. In this paper, we derive another form of QCLS problem[6,7] where all constrained matrices are symmetric and positive definite, and then solve the optimization problem by penalty function method. So the PR CM-QMF banks with high stopband attenuation can be easily obtained by the new method.

The feature comparison between existing design methods and our proposed method is tabulated in Tab.1.

The remainder of the paper is organized as follows: In Section II, we present the design problem formulation of paraunitary CM-QMF bank. In Section III, we describe our method and analysis. In Section IV, we give two design examples. Finally, Section V concludes this

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### Tab.1 Comparisons between current existing methods and our method

<table>
<thead>
<tr>
<th>Item</th>
<th>Current methods</th>
<th>Our method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial approximate solution</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Iterative manner</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Local minima</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Complex nonlinear optimization</td>
<td>Linearized</td>
<td>Yes</td>
</tr>
<tr>
<td>PR CM-QMF bank with high stopband attenuation</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

#### Notations
Bold-faced letters represent vectors and matrices. Superscript $T$ denotes transposition, and the ‘tilde’ accent on a function is defined such that $\tilde{A}(z) = A^+(1/z^*)$ where $*$ denotes the conjugation of coefficients. $I_k$ and $J_k$ denote $k \times k$ identity and reversal matrix, respectively. $\lfloor x \rfloor$ denotes the largest integer less or equal to $x$.

### II. Design of Paraunitary Cosine-Modulated QMF Banks

#### 1. Basic principle

An $M$-channel maximally decimated QMF bank is shown in Fig.1. The QMF banks are used in a wide range of speech, image, and other applications, which involve the splitting of an input signal into subbands and, finally, the reconstruction of the original signal. There are two prevenient $M$-channel QMF banks such as the perfect reconstruction QMF banks and the pseudo-QMF banks. The latter has an efficient design procedure (only the prototype filter is designed), but the former achieves perfect reconstruction of the input (i.e., without aliasing magnitude, or phase distortion).

![Fig.1 M-channel maximally decimated filter bank](image)

In a cosine-modulated QMF bank, the transfer functions of the analysis and synthesis filters $H_k(z)$ and $F_k(z)$ for $k = 0, 1, \cdots, M - 1$, respectively, are obtained by modulating the transfer function $P(Z)$ of a prototype linear-phase lowpass FIR filter with bandwidth $\pi/(2M)$. So, the impulse responses of the analysis and synthesis filters $H_k$ and $F_k$ are given by\[^2,3\]

\begin{align}
h_k(n) &= 2p(n) \cos \left( \frac{(2k + 1)\pi}{2M} \left( n - \frac{N - 1}{2} \right) + (-1)^k \frac{\pi}{4} \right) \tag{1a} \\
f_k(n) &= 2p(n) \cos \left( \frac{(2k + 1)\pi}{2M} \left( n - \frac{N - 1}{2} \right) - (-1)^k \frac{\pi}{4} \right) \tag{1b}
\end{align}