Delay-dependent H-infinity control for linear descriptor systems with delay in state

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Abstract: A delay-dependent H-infinity control for descriptor systems with a state-delay is investigated. The purpose of the problem is to design a linear memoryless state-feedback controller such that the resulting closed-loop system is regular, impulse free and stable with an H-infinity norm bound. Firstly, a delay-dependent bounded real lemma (BRL) of the time-delay descriptor systems is presented in terms of linear matrix inequalities (LMIs) by using a descriptor model transformation of the system and by taking a new Lyapunov-Krasovskii functional. The introduced functional does not require bounding for cross terms, so it has less conservatism. Secondly, with the help of the obtained bounded real lemma, a sufficient condition for the existence of a new delay-dependent H-infinity state-feedback controller is shown in terms of nonlinear matrix inequalities and the solvability of the problem can be obtained by using an iterative algorithm involving convex optimization. Finally, numerical examples are given to demonstrate the effectiveness of the new method presented.

Keywords: Delay descriptor systems; Delay-dependent; H-infinity control; Bounded real lemma (BRL); Linear matrix inequalities (LMIs)

1 Introduction

Since the late 1980s, the $H_\infty$ control theory has been developed because of its both practical and theoretical significance\cite{1-3}. A great deal of effort has been devoted to the investigation of $H_\infty$ control for time-delay systems\cite{4-7}. The main reason is that time-delay phenomenon is often encountered in control systems either in the state, the control input, or the measurements and the time-delay systems often caused instability and performance degradation of systems\cite{8-10}.

The existing results for $H_\infty$ control of time-delay systems deal with delay-independent and delay dependent. The delay-independent type is generally conservative, especially when a delay is short. In order to reduce the conservatism of delay-dependent type, there have been various approaches and some of the methods for nondescriptor systems with state-delay have been extended to descriptor system with delay.

Descriptor time-delay systems describe a broad class of systems which are not only of theoretical interest but also have great practical significance. We have found that they often appear in various engineering systems, including aircraft stabilization, chemical engineering systems, lossless transition lines, power systems, robotic systems etc\cite{11-15}. To the best of our knowledge, there are only few papers on the delay-dependent $H_\infty$ control for descriptor systems with delay. Recently, E. Fridman & U. Shaked\cite{7} reduced the conservatism of delay-dependent conditions by applying the descriptor model transformation and by taking a new Lyapunov-Krasovskii functional\cite{8}. The results are based on the so-called Park's inequality for bounding cross terms.

The main objective of the delay-dependent $H_\infty$ control in this paper is to design a memoryless state feedback controller such that the resulting closed-loop system is regular, impulse free, stable, as well as satisfying a prescribed $H_\infty$ norm bound constraint, which it allows a maximum delay size for a fixed $H_\infty$ performance bound or achieves a minimum $H_\infty$ performance bound for a fixed delay size. By using a descriptor model transformation of the system and by taking a new Lyapunov-Krasovskii functional according to Park\cite{4}, which does not require bounding for cross terms, a new delay-dependent bounded real lemma for systems with a state-delay is derived in terms of LMIs. Based on the bounded real lemma obtained, a sufficient condition...
for the existence of $H_\infty$ control is given in terms of nonlinear matrix inequality. An iterative algorithm involving convex optimization was developed to solve the nonlinear matrix inequality, which is similar to Moon et al[16].

This paper is organized as follows: the delay-dependent $H_\infty$ control problem for descriptor systems which we deal with is stated precisely in Section 2; A new bounded real lemma is obtained in Section 3; In Section 4, based on the obtained bounded real lemma, a sufficient condition for the existence of a delay-dependent $H_\infty$ state-feedback control is given in terms of nonlinear matrix inequality and the procedure to solve the problem is also detailed in the same section; Examples are given to illustrate the effectiveness we proposed about the delay-dependent $H_\infty$ control in Section 5; Conclusion is given in the last Section of this paper.

Notations Throughout the paper, the superscript $W^T$ stands for the transpose of any matrix $W$; $R$ denotes the set of real numbers; $R^n$ denotes the $n$-dimensional Euclidean space; $\| \cdot \|$ stands for the induced matrix 2-norm; $\lambda_{\text{min}}(\cdot)$ and $\lambda_{\text{max}}(\cdot)$ denote the minimum and the maximum eigenvalue of the corresponding matrix, respectively; $R^{n \times m}$ is the set of $n \times m$ real matrices; $C_\tau[a,b]$ denotes the Banach space of continuous vector functions mapping the interval $[a,b]$ into $R^n$; the space of functions in $R^n$ that are square integral over $[0,\infty)$ is denoted by $L^2_\infty[0,\infty)$; $xt(0) = x(t + 0)$, $t \in [-h,0]$, $h$ is a known positive delay; $I$ denotes an identity matrix of appropriate dimension and $*$ represents the elements below the main diagonal of a symmetric block matrix; the notation $X > 0$ (respectively, $X \geq 0$) means that the matrix $X$ is real symmetric positive definite (respectively, positive semi-definite); matrices, if not explicitly stated, are assumed to have compatible dimensions.

2 Description and preliminaries

Consider the following class of descriptor system with state delay:

\[ E\dot{x}(t) = A_0x(t) + A_1x(t-h) + B_0u(t) + B_1\omega(t), \quad (1a) \]
\[ z(t) = \begin{bmatrix} C_0x(t) \\ Du(t) \\ C_1x(t-h) \end{bmatrix}, \quad (1b) \]
\[ x(t) = 0, \quad t \leq 0, \quad (1c) \]

where $x(t) = [x_1(t) \quad x_2(t)]^T \in R^n$ is the system state vector, $x_1(t) \in R^r, x_2(t) \in R^{n-r}, u(t) \in R^l, \omega(t) \in L^2_\infty[0,\infty)$ is the exogenous disturbance signal and $z(t) \in R^p$ is the controlled output. The matrix $E \in R^{n \times n}$ may be singular, we shall assume that $\text{rank}(E) = r < n$. $A_0, A_1, B_0, B_1, C_0, C_1, D$ are known real constant matrices with appropriate dimensions. The time delay $h > 0$ is assumed to be unknown and with known bound $\bar{h}$.

For the sake of convenience, we assume that

\[ \begin{bmatrix} E \& B_1 \\ 0 \& 0 \end{bmatrix} = \begin{bmatrix} A_11 \& A_12 \\ A_3 \& A_4 \end{bmatrix}, \quad (2) \]
\[ B_1 = \begin{bmatrix} B_{11} \\ B_{12} \end{bmatrix}, \quad C_i = \begin{bmatrix} C_{i1} \\ C_{i2} \end{bmatrix}, \quad (i = 0, 1). \]

For a prescribed scalar $\gamma > 0$, define the performance index

\[ J(\omega) = \int_0^\infty \| z(t) \|^2 - \gamma^2 \omega^T(s)\omega(s) \, ds. \quad (3) \]

The singular delay system of (1a) with $u(t) = 0, \omega(t) = 0$ can be written as

\[ E\dot{x}(t) = A_0x(t) + A_1x(t-h). \quad (4) \]

Definition 1[10] 1) The singular delay system (4) is said to be regular and impulse free if the pair $(E, A)$ is regular (i.e. $\text{det}(sE - A)$ is not identically zero) and impulse free (i.e. $\text{deg}(\text{det}(sE - A)) = \text{rank}(E)$).

2) The singular delay system (4) is said to be stable if for any $\epsilon > 0$ there exists a scalar $\delta(\epsilon) > 0$ such that, for any compatible initial conditions $x(0)$ satisfying $\|x(0)\| < \delta(\epsilon)$, the solution $x(t)$ of system (4) satisfies $\|x(t)\| < \epsilon$ for $t \geq 0$. Furthermore, $\lim_{t \to \infty} x(t) = 0$, then the solution $x(t)$ of (4) is said to be asymptotically stable.

3) The singular delay system (4) is said to be admissible if it is regular, stable and impulse free. The objective of this paper is to design a memoryless state-feedback delay-dependent $H_\infty$-controller

\[ u(t) = Kx(t), \quad K \in R^{l \times n}, \quad (5) \]

such that for a given real number $\gamma > 0$, the closed-loop system (1) with controller (5) is admissible ($\omega(t) = 0$) and for all nonzero $\omega(t) \in L^2_\infty[0,\infty)$, the following inequality holds:

\[ \| G_{sw}(s) \| < \gamma \text{ i.e. } \| z(t) \|_2 < \gamma \| \omega(t) \|_2, \]

i.e. $J(\omega) < 0$, where $J(\omega)$ has the structure (3).

Proposition 1[7] Assume that $A_{04}$ is nonsingular. For $\omega(t) \in L^2_\infty[0,\infty)$, the solution to (1a) with $u(t) = 0$ and (1c) exists and is unique on $[0, t_1]$ for all $t_1 > 0$.

Lemma 1[12] If there exist matrix $U = U^T > 0$ and