Infinite horizon H-two/H-infinity control for descriptor systems: Nash game approach

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Abstract: This paper studies the infinite time horizon mixed H-two/H-infinity control problem for descriptor systems using Nash game approach. A necessary/sufficient condition for the existence of infinite horizon H-two/H-infinity control is presented in the form of two coupled algebraic Riccati equations, respectively. Finally, a suboptimal H-two/H-infinity controller design is given based on an iterative linear matrix inequality algorithm.

Keywords: Nash game approach; Descriptor system; H-two/H-infinity control; Coupled algebraic Riccati equations; Iterative linear matrix inequality algorithm

1 Introduction

Descriptor system has attracted a lot of researchers because of its comprehensive practical applications, such as power systems, social economic systems, circuits and robotics. Many efforts have been made on different subjects related to the class of systems, such as H₂ control [1–3] and H∞ control [4–6]. As we know, the H₂ control system has good performance, but it is not of robustness for uncertainty from the model perturbation for the controlled objects and the H∞ control method solves the problem of system robustness at the cost of its optimal performance. There are several approaches to solve the contraction between the two questions. This includes nonstandard Riccati equations [7], convex optimization [10] and so on.

Another important approach to solving the H₂/H∞ problem is Nash game theory [11], which gave the necessary and sufficient conditions for the existence of an H₂/H∞ optimal controller in the infinite horizon case and in finite horizon case. The implication of H₂/H∞ control problem defined in [11] is to require one to find a controller which satisfies the H∞ performance index and minimizes the H₂ cost function when the worst case disturbance appears, simultaneously. The mixed H₂/H∞ control is attractive in engineering practice such as stochastic power control in CDMA systems [12] and disturbance rejection in hard disk drives [13]. The results in [11] have been generalized to nonlinear systems [14], output-feedback H₂/H∞ control [15] and stochastic systems [16–17]. It should be noted that the advantage of using Nash game approach to solve the H₂/H∞ control problem is to optimize H₂ performance at the case where the system is subjected to the worst disturbance. However, to the best of the author’s knowledge, few results have been obtained on the H₂/H∞ control for descriptor systems using the method.

In this paper, we deal with infinite time horizon H₂/H∞ control for descriptor systems. Compared with normal system [11], the infinite horizon H₂/H∞ control of descriptor system is of more complexity because of its special system structure. Our main contributions of this paper are twofold: First, a necessary/sufficient condition for the existence of a pair of admissible solutions (u∗, w∗) to infinite horizon H₂/H∞ control is developed, respectively, which are presented in terms of algebraic Riccati equations (AREs). Second, because it is not easy to solve the two coupled AREs, a suboptimal H₂/H∞ controller design method is given on the basis of an iterative linear matrix inequality algorithm.

The remainder of this paper is organized as follows. In Section 2, some definitions and lemmas are introduced. In Section 3, the infinite horizon H₂/H∞ control problem is described for descriptor systems, where the existence conditions for the solution to this problem are derived. Section 4 gives a numerical algorithm to design suboptimal H₂/H∞ controller. Section 5 uses an example to show the effectiveness of the numerical algorithm. Section 6 gives conclusion of the paper.

Notations: A′: transpose of the matrix A;
A ≥ 0 (A > 0): A is positive semidefinite (positive definite) real matrix;
I: identity matrix;
∥z(t)∥₂,∥z(t)∥₂, (0,∞) = ∫₀^∞ z′(t)z(t)dt;
L₂([0,∞), ℝⁿ): the space of y(t) ∈ ℝⁿ satisfying
∫₀^∞ ∥y(t)∥² dt < ∞;
rank A: the rank of A;
deg det(sI – A): degree of determinant of sI – A;
C⁺: the closed right hand side complex plane;
∥·∥ denotes the spectral norm.

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2 Preliminaries

In this section, we introduce some definitions and lemmas, which are essential for the paper.

Consider the following descriptor system:

\[
\begin{align*}
E \dot{x}(t) &= Ax(t) + Bu(t), \\
y(t) &= Cx(t), \\
E x(0) &= Ex_0,
\end{align*}
\]

(1)

where \(x(t) \in \mathbb{R}^n\), \(u(t) \in \mathbb{R}^m\), and \(y(t) \in \mathbb{R}\) are its state, control input, and measurement output, respectively. \(E, A \in \mathbb{R}^{n \times n}\), \(B \in \mathbb{R}^{n \times m}\), and \(C \in \mathbb{R}^{1 \times n}\) are constant matrices. The matrix \(E\) may be singular. It is assumed that \(\text{rank } E = q \leq n\). For the sake of simplicity, we use \((E, A, B, C)\) to denote the descriptor system (1).

Remark 1 Because of the intrinsic property of descriptor systems, the initial values must be given in the form \(E x_0\) (see [18] for more details).

Definition 1 [19] The initial condition \(E x_0\) is said to be admissible to system (1), if the system has a solution \(x(t)\) with respect to \(E x_0\).

A special initial condition \(E x_0\) is admissible to system (1) if and only if

\[
\text{rank } [sE - A \quad B \quad E x_0] = \text{rank } [sE - A \quad B].
\]

(2)

An arbitrary initial condition is admissible to system (1) if and only if

\[
\text{rank } [E \quad A \quad B] = \text{rank } [sE - A \quad B].
\]

(3)

Definition 2 [20] i) System (1) is regular if \(\text{det}(sE - A)\) is not identically zero.

ii) System (1) is said to be impulse-free if \(\text{deg}(\text{det}(sE - A)) = \text{rank } E\).

iii) System (1) is said to be stable if all the roots of \(\text{det}(sE - A) = 0\) have negative real parts.

iv) System (1) is said to be admissible if it is regular, impulse-free and stable.

Remark 2 It is noted that the regularity of the pair \((E, A)\) guarantees the existence and uniqueness of a solution to the system \((E, A)\) for any admissible initial conditions.

Definition 3 [20] System (1) is stabilizable if there exists a state feedback control \(u(t) = K x(t)\), such that for any admissible initial condition \(E x_0 \in \mathbb{R}^n\), the closed-loop system

\[
E \dot{x}(t) = (A + BK) x(t), \quad E x(0) = E x_0
\]

(4)

is stable.

To guarantee the existence and uniqueness of solution to system (4) for any control input, we will hereafter assume that feedback control is confined to the set such that (4) is regular.

Definition 4 [21] Suppose \(x_{r}(t), y_{r}(t),\) and \(u_{r}(t)\) are impulse solutions, impulse output, and impulse input of system (1), respectively. If \(x_{r}(t)\) may be uniquely determined by \(y_{r}(t)\) and \(u_{r}(t)\) for any \(\tau \geq 0\), system (1) is impulse observable (or \((E, A, C)\) is impulse observable).

System (1) is impulse observable if and only if

\[
\text{rank } \begin{bmatrix} E & A \\ 0 & E \\ 0 & C \end{bmatrix} = n + \text{rank } E.
\]

(5)

Definition 5 [5] System (1) is detectable, if there exists a real matrix \(H\) of suitable dimension, such that

\[
E \dot{x}(t) = (A + HC) x(t)
\]

(6)

is stable.

System (1) is detectable if and only if

\[
\text{rank } \begin{bmatrix} sE - A \\ C \end{bmatrix} = n, \quad \forall s \in \mathbb{C}_+.
\]

(7)

Lemma 1 [22] If system (1) is detectable and impulse observable, then (1) is admissible if and only if there exists a solution \(P \in \mathbb{R}^{n \times n}\) to the generalized Lyapunov equation (GLE)

\[
\begin{cases}
E' P = P E & \geq 0, \\
A' P + P A + C' C = 0.
\end{cases}
\]

(8)

Consider the following descriptor system described by the state equations:

\[
\begin{align*}
E \dot{x}(t) &= A x(t) + B u(t), \\
y(t) &= C x(t), \\
E x(0) &= E x_0 \in \mathbb{R}^n,
\end{align*}
\]

(9)

where \(x(t) \in \mathbb{R}^n\) is the state, \(z_0(t) \in \mathbb{R}^n\) is the generalized Lyapunov equation (GLE)

\[
\begin{cases}
E' P = P E & \geq 0, \\
A' P + P A + C' C = 0.
\end{cases}
\]

(10)

Consider the following descriptor perturbed system:

\[
\begin{align*}
E \dot{x}(t) &= A x(t) + B_1 w(t) + B_2 u(t), \\
z(t) &= C x(t), \\
E x(0) &= E x_0.
\end{align*}
\]

(11)

3 Main results

Consider the following descriptor perturbed system:

\[
\begin{align*}
E \dot{x}(t) &= A x(t) + B_1 w(t) + B_2 u(t), \\
z(t) &= C x(t).
\end{align*}
\]

(12)