Three-dimensional pursuer convoy by using guidance laws

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Abstract: This paper investigates the problem of modeling and controlling pursuer convoy in three-dimensional space. The guidance laws applied for convoy, the velocity pursuit, the deviated pursuit and the proportional navigation, steer the pursuer using the rate of line-of-sight (LOS) between successive pursuers. On the basis of the differential equations for the range, the pitch angle of LOS and the yaw angle of LOS between successive pursuers, the guidance laws are proposed to derive decentralized control strategy for pursuer convoy. The results concerning the pursuer convoy are rigorously proven. Simulations are conducted to demonstrate the feasibility and effectiveness of the proposed control strategy.

Key words: Pursuer convoy; Three-dimensional space; Guidance laws; LOS

1 Introduction

The issues of convoy have been widely studied in recent years. A great variety of algorithms have been adopted to achieve the goal.

At present, vehicle convoy is a hot subject widely discussed in the two-dimensional space. In [1], a novel method for efficient rendezvous of an autonomous vehicle with a moving convoy was proposed in the presence of traffic on a highway. Geometrically interpreted adaptive control was proposed in [2] for controlling the motion of a convoy of automated vehicles. In [3], Henderson demonstrated the feasibility of utilizing PD and PID convoy control algorithms using only distance/velocity measurements to the following car. The objective of [4] is to present a control system which can control a convoy of follower vehicles for stop-and-go type of traffic. The note of [5] is concerned with the selection of control parameters to ensure string stability of a vehicle convoy system, while satisfying constraints imposed by considerations of safety and passenger comfort. By using relative position information for automated ground vehicle convoys [6], a following vehicle can replicate the path travelled by a lead vehicle while both are in motion and out of each other’s sight. A form of preview-active suspension control to improve convoy vehicle ride quality was proposed in [7], wherein dynamic response of the lead vehicle was used to generate feed forward preview control gains in addition to feedback control gains for the follower vehicle suspensions.

Multiple UAV (unmanned aerial vehicle) convoy protection was addressed in [8–9], where the authors presented time-optimal paths to provide convoy protection to stationary ground vehicles and propose a control strategy to provide convoy protection to ground vehicles moving in straight lines. The UAVs are modeled as Dubins vehicles flying at a constant altitude with bounded turning radius.

Robotic convoy [10–12] is studied in the literature. The control strategies for a wheeled mobile robot model that include the kinematical equations of motion were given in [10], where some simple control schemes for tracking a time-parameterizing path are proposed. Mamdani fuzzy controller was proposed to solve the decentralized control problem as applied to a set of units following a leader [11], whilst guaranteeing string stability of the convoy. A mathematical formulation of modeling and controlling a robotic convoy under two-dimensional plane was given in [12], where the robotic convoy is modeled by using guidance laws of the velocity pursuit, the deviated pursuit and the proportional navigation. The kinematics model for the tracking problem was derived in [13], where the velocity pursuit guidance law and the deviated pursuit guidance law were suggested for robot navigation. Proportional navigation [14–18] was studied in the literature. A method for wheeled mobile robot navigation based on the proportional navigation law was proposed in [14], where the angular velocity of the robot is proportional to the turning rate of the sight line angle that joins the robot and the goal. In [15], the pure proportional navigation was employed to navigate the interceptor to intercept the target in the wireless sensor networks. In addition, proportional navigation was defined in terms of the acceleration of the pursuer [16–18].

The paper is motivated by the study in [12]. Based on the pursuit situation in the three-dimensional space, the differential equations for the range, the pitch angle of LOS and the yaw angle of LOS are established in this paper. Combining the differential equations with guidance laws, the control laws of the following pursuer for the pitch angle, the yaw angle and the linear velocity are derived to form convoy. Different from two-dimensional vehicle con-

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voy, three-dimensional pursuer convoy is implemented in this paper. All the pursuers fly at a varying altitude in the three-dimensional space, which is different from the control strategy designed in [8–9]. Based on [12, 14], the control laws of this paper are defined in a simple way in terms of flight path angle and heading angle of the pursuer. The problem of convoy can be seen as a special case of group formation. Thus, on the basis of the study in [19], triangle formation is further implemented by using the deviated pursuit. Considering the influence of sensor noise, a Kalman filter [20] is utilized to perform the tracking task and smooth to the corrupted trajectory. Combining guidance laws with the Kalman filter, accurate convoy can be obtained in this paper.

The main contribution of this paper is to propose the guidance laws to implement pursuer convoy in the three-dimensional space. Making use of the velocity pursuit, the deviated pursuit and the proportional navigation, each following pursuer imitates its lead pursuer to derive convoy. Based on the deviated pursuit, further study is conducted on the implementation of triangle formation. Moreover, in the presence of sensor noise, pursuer convoy enhancement system that uses the Kalman filter is presented to enhance the precision of convoy process.

The remainder of this paper is organized as follows. Section 2 formulates the problem of convoy. The kinematics model of the pursuer is derived in Section 3. In Section 4, the guidance laws of the velocity pursuit, the deviated pursuit and the proportional navigation are discussed in more details. Section 5 proposes the theoretical derivation of pursuer convoy under guidance laws. A series of simulation results are given in Section 6. Finally, Section 7 is devoted to conclusion.

2 Problem formulation

In the three-dimensional space, given $N$ pursuers which are expected to move in a convoy, the aim is to design control strategy for $N - 1$ pursuers to follow the lead pursuer while keeping a constant distance from each other. Fig. 1 shows pursuer convoy in the reference coordinate system.

The lead pursuer which is denoted by $P_0$ may be autonomous controlled. $H_i(t) = (x_{i0}(t), y_{i0}(t), z_{i0}(t))$ denotes the path traveled by the lead pursuer, and $H_i(t) = (x_{i}(t), y_{i}(t), z_{i}(t))(i = 1, 2, \ldots, N - 1)$ denotes the path traveled by the following pursuer. The mathematical formulation of pursuer convoy is on the basis of the kinematics equations and the proposed guidance laws.

3 The kinematics model of the pursuer

The $i$th pursuer is denoted by $P_i$ for $i = 0, 1, \ldots, N - 1$. The pursuer $P_i$ has the following kinematics equations:

$$
\begin{align*}
\dot{x}_i &= v_i \cos \theta_i \cos \phi_i, \\
\dot{y}_i &= v_i \cos \theta_i \sin \phi_i, \\
\dot{z}_i &= v_i \sin \theta_i,
\end{align*}
$$

(1)

where $(x_i, y_i, z_i)$ are the coordinates of the pursuer $P_i$ in the reference coordinate system, $\theta_i$ and $\phi_i$ are the flight path and heading angles, and $v_i$ is the linear velocity of the pursuer $P_i$. The pursuit situation of two successive pursuers is illustrated in Fig. 2. The LOS between the pursuer $P_i$ and the pursuer $P_{i+1}$ is denoted by $L_{i,i+1}$. $\sigma_{i,i+1}$ is the pitch angle of $L_{i,i+1}$, and $\gamma_{i,i+1}$ is the yaw angle of $L_{i,i+1}$.

![Fig. 2 The pursuit situation between two successive pursuers.](image)

In this paper, the model of two successive pursuers is presented in the polar coordinates. Based on [16], the differential equations for the range, the pitch angle of $L_{i,i+1}$ and the yaw angle of $L_{i,i+1}$ are

$$
\begin{align*}
\dot{r}_{i,i+1} &= v_i \cos \theta_i \cos \sigma_{i,i+1} \cos(\phi_i - \gamma_{i,i+1}) \\
&\quad \quad + v_i \sin \theta_i \sin \sigma_{i,i+1} \\
&\quad \quad - v_{i+1} \cos \theta_{i+1} \cos \sigma_{i,i+1} \cos(\phi_{i+1} - \gamma_{i,i+1}) \\
&\quad \quad - v_{i+1} \sin \theta_{i+1} \sin \sigma_{i,i+1}, \\
\dot{r}_{i,i+1} \gamma_{i,i+1} &= -v_i \cos \theta_i \sin \sigma_{i,i+1} \cos(\phi_i - \gamma_{i,i+1}) \\
&\quad \quad + v_i \sin \theta_i \cos \sigma_{i,i+1} \\
&\quad \quad + v_{i+1} \cos \theta_{i+1} \sin \sigma_{i,i+1} \cos(\phi_{i+1} - \gamma_{i,i+1}) \\
&\quad \quad - v_{i+1} \sin \theta_{i+1} \cos \sigma_{i,i+1},
\end{align*}
$$

(2)