Decentralized $H_{\infty}$ state feedback control for large-scale interconnected uncertain systems with multiple delays

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Abstract: Decentralized $H_{\infty}$ control was studied for a class of interconnected uncertain systems with multiple delays in the state and control and time varying but norm-bounded parametric uncertainties. A sufficient condition which makes the closed-loop system decentralized asymptotically stable with $H_{\infty}$ performance was derived based on Lyapunov stability theorem. This condition is expressed as the solvability problem of linear matrix inequalities. The method overcomes the limitations of the existing algebraic Riccati equation method. Finally, a numerical example was given to demonstrate the design procedure for the decentralized $H_{\infty}$ state feedback controller.

Key words: uncertainty; time-delay; linear matrix inequality; decentralized $H_{\infty}$ control

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1 INTRODUCTION

In recent years, decentralized control for interconnected large-scale systems with time-delay has been paid much attention to\cite{1-9}. Since the models often contain uncertainties, the expected performance cannot be obtained if the controller is designed only based on nominal model. Moreover, there is time-delay occurring in the system matrices due to the transferring information and insensitivity to the measurement. At present, the controller design is focused on Riccati equalities or inequalities, which require pre-selection of many parameters. There exists computational complicity and inconvenient implement. Linear matrix inequality (LMI) method has been paid much attention to for its high solvability and becomes an effective method for robust analysis and synthesis\cite{10,11}.

Decentralized robust $H_{\infty}$ control for uncertain interconnected large systems with multiple delays has drawn a lot of attention\cite{12-14}. The problem of decentralized robust stabilization and robust $H_{\infty}$ performance for a class of interconnected systems with unknown delays and norm-bounded parametric delays were studied in Ref. [12]. The decentralized dynamic output feedback controllers based on state observer are given in the form of Riccati equation. The decentralized robust $H_{\infty}$ control via state feedback for uncertain interconnected linear large-scale system with multiple time-varying delays in the input, state and interconnected matrices, which overcomes the drawbacks of Riccati approach. Based on the existence of solution to the LMI, a decentralized $H_{\infty}$ state feedback controller is designed to make closed-loop system stable with the $H_{\infty}$ performance bound.

2 PROBLEM DESCRIPTION AND LEMMAS

Consider a class of time-varying uncertain time delay large-scale systems with $N$ subsystems. The subsystems $(L_i)$ can be described as follows,

\[
\begin{align*}
\dot{x}_i(t) &= [A_i + \Delta A_i(t)]x_i(t) + [A_{i1} + \Delta A_{i1}(t)]x_i(t-d_i) + [B_i + \Delta B_i(t)]u_i(t) + [B_{i1} + \Delta B_{i1}(t)]u_i(t-h_i) + \\
&\quad \sum_{j=1}^{N} A_{ij}x_j(t-r_{ij}) + B_{ij}\omega_i(t) \\
&= C_i x_i(t) + D_{i1}u_i(t) \\
x_i(t) &= \xi_i(t), t \in [-\max(d_i, h_i, r_{ij}), 0]
\end{align*}
\]

where $i, j = 1, 2, \cdots, N$, $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{r_i}$, $\omega_i(t) \in \mathbb{R}^{r_{i1}}$, and $z_i(t) \in \mathbb{R}^{p_i}$ are the state, control, dis-
turbance and controlled output vectors, respectively. \( A, B, C, D, A_t, B_t \) and \( B_o \) represent, input and output matrices with appropriate dimensions the system, respectively. \( \Delta A(t), \Delta B(t), \Delta A_t(t) \) and \( \Delta B_t(t) \) are of uncertainties on the state and input, which are continuous function of time. \( A_d \) is the interconnections. \( d, h \) and \( \tau_g \geq 0 \) denote the delay in the state, control and interconnections, respectively. \( \xi_i(t) \in \mathbb{C}^{n} \) \([-\max(d, h, \tau_g), 0]\) is the continuous initial vector function.

Suppose that
\[
\left\{ \begin{array}{l}
[\Delta A(t), \Delta B(t)] = L F_i(t)[E_i, N_i] \\
\Delta A(t) = L_1 F_i(t) E_i \\
\Delta B(t) = L_2 F_i(t) N_i
\end{array} \right.
\]  
(2)

where \( L_1, L_2, E_i, N_i, N_o \) are known, real and constant matrices, \( F_i(t) \) are unknown time-varying matrices whose elements are Lebesgue measurable and satisfy
\[
F_i^T F_i \leq I
\]  
(3)

Decentralized \( H_{\infty} \) Control

Given a constant \( \gamma > 0 \), the whole large-scale interconnected time-delay system (1) is asymptotically stable with the \( H_{\infty} \) performance \( \gamma \) if there exist local memoryless state feedback laws \( u_i(t) = K_i x_i(t) \) \( (i = 1, 2, \cdots, N) \) such that the resulting closed-loop system satisfies the following conditions:

1) The closed-loop system is asymptotically stable whenever \( \omega_i = 0 \).

2) Subject to the assumption of zero initial conditions, the following inequality holds
\[
\| z_i(t) \|_2 < \gamma \| \omega_i \|_2
\]  
(4)

for all \( \omega_i \neq 0 \) and for all admissible uncertainties.

In the following discussion, define a two-valued function \( \delta(\cdot) \) as
\[
\delta(E) = \begin{cases} 0, & E = 0 \\ 1, & E \neq 0 \end{cases}
\]

Lemma 4.4 Suppose \( X, Y \) and \( Z \) are vectors or matrixes with suitable dimensions, then for any positives \( \alpha \) and \( \beta \), we have
\[
X^T Y + Y^T X \leq \alpha X^T X + \frac{1}{\alpha} Y^T Y,
\]
\[
2Z^T Z \leq \beta Z^T Z + \frac{1}{\beta} Y^T Y.
\]

3 DECENTRALIZED ROBUST \( H_{\infty} \) CONTROL

In this section, the sufficient condition for existence of controller for large-scale uncertain systems with time-delay is derived. Then, a convex optimization problem with LMI constraints is formulated to design a decentralized \( H_{\infty} \) state feedback controller for system (1).

Theorem 1 For large-scale system (1), if there exist positive definite matrices \( P_i, H_i \in \mathbb{R}^{n_i \times n_i}, n_i = \sum_{i=1}^{N} n_i \), and positive number \( \gamma \) for admissible uncertainties, and the following LMI (\( i = 1, 2, \cdots, N \))

\[
\begin{bmatrix}
\bar{Z} & PA_{ii} & \cdots & PA_{iN} & PB_i \\
PA_{ii} & -\delta(A_{ii}) H_i & \cdots & \cdots & \cdots \\
\vdots & \cdots & \cdots & \cdots & \cdots \\
PA_{iN} & \cdots & \cdots & \cdots & -\delta(A_{iN}) H_N \\
B_i & B_i & \cdots & \cdots & -\gamma^T I
\end{bmatrix} < 0
\]  
(5)

where \( \bar{Z} = \bar{A} + (C + D K)^T (C + D K) \),
\[
\bar{A}_i = (A_i + B K_i)^T P_i + P_i (A_i + B K_i) + \\
P_i L_i^T P_i + (E_i + N K_i)^T (E_i + N K_i) + \\
P_i A_i A_i^T P_i + I + P_i L_i L_i^T P_i + E_i^T E_i + \\
P_i B_i B_i^T + K_i K_i + P_i L_i L_i^T P_i + \]
\[
K_i N_i N_i K_i + \sum_{i=1}^{N} \delta(A_i) H_i
\]
then the closed-loop system is asymptotically stable with the \( H_{\infty} \) performance \( \gamma \).

Proof 1) Asymptotical stability. Suppose that \( \omega_i = 0 \). When state feedback control \( u_i(t) = K_i x_i(t) \) \( (i = 1, 2, \cdots, N) \) is applied, the closed-loop subsystem becomes
\[
L_{ij} x_{ij}(t) = \left[ A_{ij} + \Delta A_i(t), K, x_i(t) \right]
\]
\[
\left[ A_{ij} + \Delta A_i(t), K, x_i(t - d_{ij}) \right] + \\
\left[ B_{ij} + \Delta B_i(t), K, x_i(t) \right] + \\
K_i x_i(t - h_{ij}) + \sum_{i=1}^{N} A_i x_i(t - \tau_g)
\]  
(6)

Consider the following Lyapunov function:
\[
V(x) = \sum_{i=1}^{N} \left\{ x_i^T P_i x_i + \right.
\]
\[
\int_{t-d_{ij}}^{t} x_i^T(s) \left[ I_i + E_i^T E_i \right] x_i(s) ds + \\
\int_{t-h_{ij}}^{t} x_i^T(s) \left[ K_i^T (I_i + N_i^T N_i) K_i \right] x_i(s) ds + \\
\sum_{i=1}^{N} \delta(A_i) x_i^T (t - \tau_g) H_i x_i (t - \tau_g)
\]  
(7)

where \( P_i, H_i \) are positive definite matrices. The time deviation of \( V(x) \) along any state trajectory of the system (6) satisfies
\[
V(x) = \sum_{i=1}^{N} \left\{ x_i^T P_i x_i + x_i^T (I_i + E_i^T E_i) x_i - x_i^T (t - d_{ij}) (I_i + E_i^T E_i) x_i (t - d_{ij}) + \\
x_i^T \left[ K_i^T (I_i + N_i^T N_i) K_i \right] x_i + \sum_{i=1}^{N} \delta(A_i) x_i^T H_i x_i - \\
x_i^T (t - h_{ij}) \left[ K_i^T (I_i + N_i^T N_i) K_i \right] x_i (t - h_{ij}) - \\
\sum_{i=1}^{N} \delta(A_i) x_i^T (t - \tau_g) H_i x_i (t - \tau_g) \right\}
\]
\[
= \sum_{i=1}^{N} \left\{ x_i^T \left[ (A_i + B K_i)^T P_i + P_i (A_i + B K_i) + \\
(\Delta A_i + \Delta B_i K_i)^T P_i + P_i (\Delta A_i + \Delta B_i K_i) \right] x_i + \\
2 x_i^T P_i \left[ (A_i + \Delta A_i) x_i (t - d_{ij}) + 2 x_i^T P_i (B_i + \Delta B_i K_i) x_i (t - h_{ij}) + \\
\sum_{j=1}^{N} 2 x_j^T P_i A_j x_j (t - \tau_g) + \\
x_i^T (I_i + E_i^T E_i) x_i - x_i^T (t - d_{ij}) (I_i + E_i^T E_i) x_i (t - d_{ij}) - \\
x_i^T (t - h_{ij}) \left[ K_i^T (I_i + N_i^T N_i) K_i \right] x_i (t - h_{ij}) - \\
\sum_{i=1}^{N} \delta(A_i) x_i^T H_i x_i \right\}
\]