Fuzzy controller based on chaos optimal design and its application

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Abstract: In order to overcome difficulty of tuning parameters of fuzzy controller, a chaos optimal design method based on annealing strategy is proposed. First, apply the chaotic variables to search for parameters of fuzzy controller, and transform the optimal variables into chaotic variables by carrier-wave method. Making use of the intrinsic stochastic property and ergodicity of chaos movement to escape from the local minimum and direct optimization searching within global range, an approximate global optimal solution is obtained. Then, the chaos local searching and optimization based on annealing strategy are cited, the parameters are optimized again within the limits of the approximate global optimal solution, the optimization is realized by means of combination of global and partial chaos searching, which can converge quickly to global optimal value. Finally, the third order system and discrete nonlinear system are simulated and compared with traditional method of fuzzy control. The results show that the new chaos optimal design method is superior to fuzzy control method, and that the control results are of high precision, with no overshoot and fast response.

Key words: fuzzy controller; chaos algorithm; parameter; optimal control

1 INTRODUCTION

Fuzzy control is a no-model control method based on knowledge. It has properties of simplicity and flexible application, easy implementation, strong robustness. Therefore, it is a powerful tool to study complicate problems and has been widely applied in complex industrial systems in which it is difficult to construct a precise mathematical model and the parameters are uncertain. However, fuzzy control also has its disadvantages, for example, it is hard to obtain the optimal membership functions and control rules. And the control results often show strong overshoot and oscillation, especially when the processes are of highly nonlinear and with high dimensions, the optimal parameters are acquired by trial-and-error methods of expert’s experiences, which takes lots of time and is difficult to obtain optimal fuzzy system[1-3]. In order to get optimal parameters of fuzzy controller, a chaos optimal algorithm based on annealing strategy is proposed. Due to ergodic and random features of chaos dynamics, it can continually search all conditions according to its rule within ranged space. First, the optimal variables are transformed into chaotic variables that are applied to global searching and optimization for parameters of fuzzy controller, an approximate global optimal solution is obtained. And then, the chaos local searching and optimization based on annealing strategy are cited, the parameters are optimized again, and the optimization is realized by means of combination of global and partial chaos searching, which has higher ability of searching for global optimal solutions[4-9].

2 REALIZATION OF FUZZY CONTROLLER

The method of fuzzy inference in Ref. [10] is adopted, NB stands for negative big, PB for positive big and AZ for approximate zero. Triangular membership functions are applied to express the membership functions for the input E and output Ur of fuzzy controller [11-13]. When the membership functions for E are fixed (E is "the normalized value of error signal e(k), \(-1 \leq E \leq 1\)"), the membership functions for Ur will change according to the parameters x1, x2 (x1 and x2 are structure parameters of the controller), and are shown in Fig. 1.

Defining

\[ E_4 = \frac{x_1 - x_2}{1 + x_1 - x_2} \] (1)

the three different ranges determined by \( E_4 \) are shown in Fig. 2.

According to Figs. 1 and 2, the fuzzy control algorithm is described as follows.

Case 1 \( 0 < x_1 < x_2 < 1 \)

\[ u_p = \frac{E_4 \cdot E_3 \left[ 3x_1 (2 - |E|) + E_5 (3 - E^3) \right]}{3 \left[ 2x_1 (1 - E^2) + E_5 (2 |E| - E^3) \right]} \] (2)

Case 2 \( 0 < x_2 < x_1 < 1 \)

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Foundation item: Project (01JJY3029) supported by the Natural Science Foundation Hunan Province
Received date: 2003-05-06; Accepted date: 2003-07-28

CLC number: TP271+.72 Document code: A


Vol. 11 No. 1 J. CENT. SOUTH UNIV. TECHNOL. March 2004

Foundation item: Project (01JJY3029) supported by the Natural Science Foundation Hunan Province
Received date: 2003-05-06; Accepted date: 2003-07-28

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1) When $0 < |E| < E_d$, 
$$u_f = \frac{E^2(3(1-x_1^2)+3x_1^2 |E| - x_1^2 E^2)}{3[2x_1+2(1-x_1) |E| - x_1^2 E^2]} \quad (3)$$

2) When $0 < |E| < 1 - E_a$, 
$$u_f = E_E [E_E [3x_2(2-|E|)+E_E(3-E^2)] - E_E E_E [(3-E_a) x_1 - E_E)]] / (3|E|[2x_1(1-E^2)+E_E(2|E|-E^2)-E_E E_E]) \quad (4)$$

3) When $1 - E_d < |E| < 1$, 
$$u_f = E_E [E_E [3-x_1 (1+|E|+E^2)] - E_E E_E [(3-E_a) x_1 - E_E)]] / (3|E|[E_E (2+x_1 E_E) - E_E E_E]) \quad (5)$$

where $E_1 = 1 - |E|, E_2 = 1 + |E|, E_3 = 1 - 2|E|$, $E_4 = x_1 - x_2, E_5 = 1 - x_2$.

![Fig. 1 Membership function of fuzzy controller](image)

![Fig. 2 Three ranges with respect to $E_d$](image)

3 CHAOS OPTIMIZATION ALGORITHM

The structure parameters $x_1, x_2$ of fuzzy controller are required to optimize. Suppose that the expected output is $r_e$ and the system output is $y_e$, the object function can be defined as follows:

$$J = \frac{1}{2} \sum_{i=1}^{N} (r_e - y_e)^2 \quad (6)$$

The constraint condition is as follows:

$$0 < x_1, x_2 < 1$$

Consider Logistic mapping:

$$q_{n+1} = \mu q_n (1-q_n), n = 0, 1, 2, \ldots, N, q_0 \in (0, 1) \quad (7)$$

where $q = (q_{1,n}, q_{1,n}, \ldots, q_{1,n})$ are the numbers of chaotic variables, $N$ is the frequency of chaos learning, $\mu$ is control parameter. When $\mu = 4$, the mapping is perfect one at $(0, 1)$ intervals and the system is in full chaotic state, $q$ are ergodic at $(0, 1)$ intervals and generate chaotic sequence. Since chaotic variables are extremely sensitive to the initial value, fetching some initial values with tiny discrepancy to Eqn. (7) can get a sequence of chaotic variables with different traces. When the chaotic variables are carried-wave and iterated, respectively, the current optimal parameters (global sub-optimization solutions) $x_i^* (i=1, 2)$ are obtained. And then, the method of chaos partial searching based on annealing strategy is applied, which automatically reduces the range of chaos searching and implements optimal method of combining global and partial searching. Therefore, the searching speed is accelerated and the parameters are converged to global optimal solution.

Based on the above analysis, the following equations can be obtained:

$$x_{n+1} = \mu x_n (1-x_n), \mu = 4, x_n \in (0, 1) \quad (8)$$

$$x_{1,n}^* = x_{1,n} + z(t) x_n \quad (9)$$

$$x_{2,n}^* = x_{2,n} + z(t) x_n \quad (10)$$

where $n = 0, 1, \ldots, N, z(t)$ is time parameter based on annealing strategy, $\alpha$ is the decay factor of $z(t)$.

The optimization process can be described as the following steps\cite{14-18}.

**Step 1** Algorithm initialization, $n = 0, t = 0$. Fetch two different initial values randomly at $(0, 1)$ intervals and substitute in Logistic mapping of Eqn. (8), respectively, two chaotic variables $x_{1,n}, x_{2,n}$ are obtained. According to the system demand, each chaotic variable $x_{1,n}$ at $(0, 1)$ intervals is mapped to correspondent intervals.

**Step 2** The chaotic variables are substituted in the parameters $x_1, x_2$ of Eqns. (2)-(5), according to Eqn. (6), let $J^* = J(0), x_i^* = x_{i,n}$, calculate the object function $J$.

**Step 3** When $n < N$, proceed to the next step, otherwise go to step 5.

**Step 4** If $J(n) < J^*$, then $J^* = J(n), x_i^* = x_{i,n}^*$, if $J(n) \geq J^*$, then abandon $x_{i,n}, n = n + 1$, go to step 2.

**Step 5** $t = t + 1$, the global suboptimization solutions $x_i^*$ are substituted in Eqns. (9) and (10) to calculate global optimal values $x_i^*$.

**Step 6** $z(t)$ is calculated according to Eqn. (11), if $z(t) < Z$ (searching closing condition), then proceed to the next step, otherwise go to step 5.

**Step 7** The global optimal solutions $x_i^*$ ($i = 1, 2$) are obtained.

4 SIMULATION

Initial values of chaotic variables are set to be random numbers at $(0, 1)$ intervals, chaotic variables $x_{1,n}$ of Eqns. (7)-(10) are mapped at $(-0.5, 0.5)$ intervals, and $\alpha = 0.01, N = 1 \times 10^4, z(0) = 0.4, Z = 1 \times 10^{-1}$.

**Example 1** Control plant is the third order delay system as follows:

$$G(s) = \frac{Ke^{-\alpha}}{7.6s^3 + 12.3s^2 + 5.4s + 1} \quad (12)$$

The sampling period is 0.2 s.