Control of acrobot based on Lyapunov function

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Abstract: Fuzzy control based on Lyapunov function was employed to control the posture and the energy of an acrobot to make the transition from upswing control to balance control smoothly and stably. First, a control law based on Lyapunov function was used to control the angle and the angular velocity of the second link towards zero when the energy of the acrobot reaches the potential energy at the unstable straight-up equilibrium position in the upswing process. The controller based on Lyapunov function makes the second link straighten naturally relative to the first link. At the same time, a fuzzy controller was designed to regulate the parameters of the upper control law to keep the change of the energy of the acrobot to a minimum, so that the switching from upswing to balance can be properly carried out and the acrobot can enter the balance quickly. The results of simulation show that the switching from upswing to balance can be completed smoothly, and the control effect of the acrobot is improved greatly.

Key words: acrobot; fuzzy control; Takagi-Sugeno fuzzy model; model-free fuzzy control

1 INTRODUCTION

An acrobot is a two-link manipulator operating in a vertical plane with an actuator at the elbow but no actuator at the shoulder[1,2]. A great number of studies have devoted to the control of an acrobot. To keep the control law simple, the motion space is usually divided into two subspaces, i.e., upswing subspace and the balance subspace. For example, Hauser et al[3] investigated the problem of balancing an acrobot at the unstable straight-up equilibrium position using nonlinear approximation. In Refs. [4, 5], Spong described a partial feedback linearization method to swing an acrobot up and used the linear quadratic regulator (LQR) method to balance it. Theoretically, it did not guarantee the energy of the acrobot to increase with each swing. In addition, the LQR balancing control law[4, 5] makes the region for balance control very small. Other control methods[6-9] are complicated and/or have a relatively long settling time. In Ref. [10], LAI et al presented a fuzzy control strategy for an acrobot that combines model-free and model-based fuzzy controllers to control upswing and balance subspace, respectively. The control strategy is very simple and the control results are quite satisfactory. However, there is no guarantee for acrobot switching from upswing to balance.

In this paper, it is found that the switching from upswing to balance may cause a serious problem if not handled properly. To solve this problem, a fuzzy control method based on Lyapunov function was designed to guarantee that the switching is carried out stably, smoothly and quickly. The controller straightens the links of the acrobot and reduces the change of the energy to a minimum. This switching strategy enables the acrobot to switch from upswing to balance smoothly, and then to be stabilized easily in the balance subspace.

2 CONTROL LAWS FOR UPSWING AND BALANCE

Due to the complex nonlinear characteristics of the acrobot, the motion space is usually divided into two subspaces: the upswing subspace and the balance subspace. The balance subspace is in the neighborhood of the unstable straight-up equilibrium position, and the upswing subspace is the remainder. A model-free and a model-based fuzzy control laws were employed in the upswing and balance subspaces, respectively. The control laws for upswing and balance are presented as follows.

2.1 Dynamics of acrobot

The model of an acrobot is shown in Fig. 1. Let \( q = [q_1, q_2]^T \) be the joint angles. Their dynamics is given by

\[
M(q) \ddot{q} + h(q, \dot{q}) + g(q) = \tau \tag{1}
\]

where \( M(q) \) is the inertia matrix, \( h(q, \dot{q}) \) is the combination of the coriolis and centrifugal forces.
\( g(q) \) is the gravity force, and \( \tau \) is the vector of actuator torque. All of them are defined as follows.

\[
M(q) = \begin{bmatrix} m_{11}(q) & m_{12}(q) \\ m_{21}(q) & m_{22}(q) \end{bmatrix} \quad (2a)
\]

\[
h(q, q) = \begin{bmatrix} h_1(q, q) \\ h_2(q, q) \end{bmatrix} \quad (2b)
\]

\[
g(q) = \begin{bmatrix} g_1(q) \\ g_2(q) \end{bmatrix} \quad (2c)
\]

\[
\tau = [0 \tau_2]^T \quad (2d)
\]

The components of \( M(q), h(q, q) \) and \( g(q) \) are as follows.

\[
m_{11}(q) = m_1L_1^2 + I_1 + m_2L_2^2 + I_2 + 2m_2L_1L_2 \cos q_2 + m_2L_2^2;
\]

\[
m_{12}(q) = m_{21}(q) = m_2L_2^2 + I_1 + m_2L_1L_2 \cos q_1;
\]

\[
m_{22}(q) = m_2L_2^2 + I_1;
\]

\[
h_1(q, q) = -m_2L_1L_2(\dot{q}_1 \dot{q}_2 + \dot{q}_1^2) \sin q_2;
\]

\[
h_2(q, q) = m_2L_1L_2 \dot{q}_1 \dot{q}_2 \sin q_1;
\]

\[
g_1(q) = -(m_1L_1 + m_2L_1) g \sin q_1 - m_2L_2 \dot{q}_1 g \sin (q_1 + q_2);
\]

\[
g_2(q) = -m_2L_2 \dot{q}_2 g \sin (q_1 + q_2).
\]

For the link \( i = 1, 2 \) the parameters \( q_i, \dot{q}_i, m_i, L_i, L_g \) and \( I_i \) are the angle, the angular velocity, the mass, the link length, the center of mass, and the moment of inertia around the center of gravity, respectively.

The motion space of the acrobot is divided into two subspaces according to Definition 1.

**Definition 1** Two small positive numbers, \( \lambda_1 \) and \( \lambda_2 \), are used for the definition of the two subspaces.

**Upswing subspace:**

\[
|q_1| > \lambda_1 \text{ or } |q_1 + q_2| > \lambda_2 \quad (3a)
\]

**Balance subspace:**

\[
|q_1| \leq \lambda_1 \text{ and } |q_1 + q_2| \leq \lambda_2 \quad (3b)
\]

\[\text{○—Actuated joint}
\]

\[\text{▲—Passive joint}\]

**Fig. 1** Model of acrobot

### 2.2 Control law in upswing subspace

The key point in the upswing subspace is to increase the energy of the acrobot as fast as possible so that the acrobot quickly moves into the balance subspace. For this purpose, the control torque is derived directly from the energy of the acrobot. A model-free fuzzy controller is designed to regulate the amplitude of the control torque according to the energy. It is employed until the energy reaches a prescribed value, which is the potential energy of the unstable straight-up equilibrium position. The main feature of the model-free fuzzy controller is that the amplitude of the control torque decreases as the energy increases. Therefore, it guarantees the smoothness when the control strategy switches.

The energy of the acrobot is given by

\[
E(q, \dot{q}) = \frac{1}{2} \dot{q}^T M(q) \dot{q} + m_1 g L_1 \cos q_1 + m_2 g L_2 \cos (q_1 + q_2) + L_1 \cos q_1 \quad (4)
\]

To guarantee that the energy increases with each swing in the swing-up area, the first derivative of the energy should satisfy the condition

\[
E(q, \dot{q}) \geq 0 \quad (5)
\]

A simple calculation shows that the derivative of the energy with respect to time is

\[
\dot{E}(q, \dot{q}) = \dot{q}_1 \quad (6)
\]

The control torque for swing-up is chosen to be

\[
r_2 = \text{sgn}(\dot{q}_1) \cdot v, \quad v \geq 0 \quad (7)
\]

to satisfy Eqn. (5), where \( \text{sgn}(\cdot) \) is the signum function.

The control variable in Eqn. (7) can be chosen arbitrarily in the admissible range of the control torque as long as it is positive. Clearly, the amplitude of the control torque should be chosen so that it decreases as the energy increases, which allows the acrobot to move smoothly when the control law changes. This strategy is implemented by using the fuzzy rules as follows.

- If \( E(q, \dot{q}) \) is small, then \( v \) will be large.
- If \( E(q, \dot{q}) \) is medium, then \( v \) will be medium.
- If \( E(q, \dot{q}) \) is large, then \( v \) will be small.

### 2.3 Control law in balance subspace

The dynamics in the balance subspace is nonlinear. A linear approximate model around the unstable straight-up equilibrium position is usually used for the design of control law. To achieve better control, a Takagi-Sugeno fuzzy model is used to describe the nonlinearity in the subspace more precisely. The dynamics in the subspace are first captured by a set of fuzzy implications that characterize local relations using local linear approximate models. Then, a set of local controllers is designed based on those local linear approximate models. Finally, the fuzzy controller obtained by fuzzy blending of the local controllers is used to balance the acrobot. The stability of the fuzzy control system for balance is guaranteed by a set of linear matrix inequalities (LMIs).

In balance area, the Takagi-Sugeno fuzzy model is given by the following rules.

**Rule 1** If \( z \) is larger than \( c_1 \), then \( \dot{x} = A_1 x + B_1 \tau_2 \).