Design of decentralized robust $H_\infty$ state feedback controller

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Abstract: The design of decentralized robust $H_\infty$ state feedback controller for large-scale interconnected systems with value bounded uncertainties existing in the state, control input and interconnected matrices was investigated. Based on the bounded real lemma a sufficient condition for the existence of a decentralized robust $H_\infty$ state feedback controller was derived. This condition is expressed as the feasibility problem of a certain nonlinear matrix inequality. The controller, which makes the closed-loop large-scale system robust stable and satisfies the given $H_\infty$ performance, is obtained by the offered homotopy iterative linear matrix inequality method. A numerical example is given to demonstrate the effectiveness of the proposed method.

Key words: value bounded uncertainty; decentralized control; robust $H_\infty$ control; nonlinear matrix inequality; homotopy iterative method

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1 INTRODUCTION

Decentralized control has become an important branch of the large-scale system theory because of the reliability, real time and economy of its implementation\(^1\). In general, since the models often contain uncertainties, expected performance can not be obtained if the controller designed is only based on nominal model. Thus it is desired that control system design be able to take into account modeling errors as well as parameter variations in the system. In recent years, decentralized robust $H_\infty$ control problems for large-scale interconnected systems with norm bounded uncertainties or satisfying the matching conditions uncertainties have been paid much attention to\(^2\-^7\). The uncertainty is often value bounded which is more universal and need not satisfy the so-called matching conditions in practical engineering\(^8\)\-^\(^1\). The robust control problem for system with value bounded uncertainties has given rise to a lot of attention, and the results mainly focus on the robust stabilization, robust tracking and robust fault-tolerant\(^9\)\-^\(^12\). However, there is no relative report about decentralized robust $H_\infty$ control for the large-scale systems with value bounded uncertainties. In this study linear matrix inequality (LMI) approach\(^13\) is used to investigate the decentralized robust $H_\infty$ control problem for interconnected large-scale systems with value bounded uncertainties existing in the state, control input and interconnected matrices. Based on the bounded real lemma\(^14\) a sufficient condition for the existence of a decentralized robust $H_\infty$ state feedback controller is derived as the feasibility problem of a certain nonlinear matrix inequality (NLMI). The controller, which makes the closed-loop large-scale system robust stable and satisfies the given $H_\infty$ performance, is obtained iteratively by choosing suitable homotopy function\(^15\)\-^\(^16\).

2 PROBLEM DESCRIPTION

Consider the following uncertain large-scale interconnected system composed of $N$ subsystems

$$\dot{x}_i(t) = (A_{i} + AA_{j})x_i(t) + NB_{i}e_0_i(t) + (B_{zi} + AB_{zi})u_i(t) + 2(A_{ii} + AA_{ij})x_j(t)$$

$$Z_i(t) = C_{i}x_i(t) + D_i e_0_i(t)$$

$$y_i(t) = x_i(t)$$

(1)

where $i = 1, 2, \cdots, N_j$, $x_i(t) \in \mathbb{R}^n$, $e_0_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^m$, $Z_i(t) \in \mathbb{R}^l$, and $y_i(t) \in \mathbb{R}^q$ are the state, disturbance input, control input, controlled output, and measured output vectors of subsystem $i$, respectively. The matrices $A_i$, $B_{i}$, $B_{zi}$, $C_{i}$, $D_{i}$, and $D_{zi}$ are constant and of appropriate dimensions. The matrix $A_q$ describes the interconnection from subsystem $j$ to subsystem $i$. The matrices $\Delta A_i$, $\Delta B_i$ and $\Delta A_{ij}$ denote value bounded uncertainties\(^6\) in the state, control input and interconnection matrices. We suppose that

$$|\Delta A_i| < R_i$$

$$|\Delta B_i| < S_i$$

(2)

where $i = 1, 2, \cdots, N_j$, $j = 1, 2, \cdots, N_i$, $R_{i}$ and $S_{i}$ are nonnegative constant matrices with the same
dimensions as $\Delta A_j$ and $\Delta B_z$, respectively. The meanings of $E_i \preceq E$ is that $e_{ij} \preceq e_{ij}$ ($i=1, 2, \cdots, N_1$, $j=1, 2, \cdots, N_2$), where $e_{ij}$ and $e_{ij}$ are the corresponding elements of matrices $E$ and $E$ respectively whose subscript is $(i, j)$.

The whole large-scale interconnected system is written as follows:

$$
\begin{align}
\dot{x} &= (A + \Delta A)x + B_1 \omega + (B_2 + \Delta B_2) u \\
\dot{z} &= C_1 x + D_1 \omega + D_2 u \\
y &= x
\end{align}
$$

where

$$
A = [A_i]_{N \times N}, \quad \Delta A = [\Delta A_i]_{N \times N}, \\
B_1 = \text{block\_diag}(B_{11}, \cdots, B_{1N}), \\
B_2 = \text{block\_diag}(B_{21}, \cdots, B_{2N}), \\
\Delta B_2 = \text{block\_diag}(\Delta B_{21}, \cdots, \Delta B_{2N}), \\
C_1 = \text{block\_diag}(C_{11}, \cdots, C_{1N}), \\
D_1 = \text{block\_diag}(D_{11}, \cdots, D_{1N}), \\
D_2 = \text{block\_diag}(D_{21}, \cdots, D_{2N}), \\
x = [x_1, \cdots, x_{N_1}], \omega = [\omega_1, \cdots, \omega_{N_2}], \\
u = [u_1, \cdots, u_{N_2}], z = [z_1, \cdots, z_{N_1}], \\
y = [y_1, \cdots, y_{N_2}].
$$

For large-scale interconnected system (1), a state feedback decentralized $H_\infty$ control law is

$$
u = K_d y = K_d x
$$

where

$$K_d = \text{block\_diag}(K_1, \cdots, K_i, \cdots, K_N) \in \Phi
$$

herein $\Phi$ is the set of block diagonal matrices given by local state feedback control from the subsystems, $K_i \in \mathbb{R}^{m_i \times n_i}$ ($i=1, 2, \cdots, N$).

To guarantee the existence of a decentralized $H_\infty$ controller, we assume that system (1) has no unstable decentralized fixed eigenvalue under the control law (4) and the control structure (5). Denoting by $T_{wo}(K_d)$ the transfer function from the disturbance input $\omega$ to the controlled output $z$, then

$$
T_{wo}(K_d) = (C_1 + D_2 K_d)(sI - A - \Delta A - B_1 K_d - \Delta B_2 K_d)^{-1} B_1 D_1
$$

Decentralized robust $H_\infty$ state feedback control problem: For large-scale system (1), find a decentralized feedback matrix $K_d \in \Phi$, such that the resulting closed-loop system is asymptotically stable for all uncertainties Eqn. (2), and satisfies $\|T_{wo}(K_d)\| < \gamma$, where $\gamma > 0$ is a pre-specified constant.

Lemma 1 Suppose that matrices $A, B \in \mathbb{R}^{r \times s}$ and $A \geq B$. Then, for any matrix $C \in \mathbb{R}^{r \times s}$, the inequality $C^T \Delta A C \geq C^T B C$ holds.

Lemma 2 Suppose that $X$ and $Y$ are matrices or vectors of appropriate dimensions, then for any $a > 0$, the inequality $X^T Y + Y^T X \preceq a X^T X + a^{-1} Y^T Y$ holds.

Lemma 3 Suppose that $\Delta A$ is an $n \times m$ dimension matrix which satisfies $|\Delta A| < D$, the inequalities $\Omega(D) \geq \Delta A \Delta A^T$, $\Gamma(D) \geq \Delta A^T \Delta A$ holds,

$$
\Omega(D) = \begin{bmatrix} DD^T & I \end{bmatrix}, \quad \Gamma(D) = \begin{bmatrix} I \end{bmatrix} \begin{bmatrix} DD^T \end{bmatrix} D
$$

$$
\begin{cases}
\Omega(D) = \begin{bmatrix} I \end{bmatrix} \begin{bmatrix} DD^T \end{bmatrix}, & \text{otherwise} \\
\Gamma(D) = \begin{bmatrix} I \end{bmatrix} \begin{bmatrix} D^T D \end{bmatrix}, & \text{otherwise}
\end{cases}
$$

herein $\Delta A = \text{diag}(r_1, r_2, \cdots, r_m)$, $R = (r_q)$ real symmetric matrix of order $n$.

3 DECENTRALIZED ROBUST $H_\infty$ CONTROLLER DESIGN

In this section, a sufficient condition for the existence of a decentralized robust $H_\infty$ state feedback controller for the system (1) is presented, and an iterative LMI algorithm based on homotopy method is put forward to obtain decentralized $H_\infty$ state feedback controller.

Theorem 1 For the large scale system (1), if there exist positive numbers $a > 0$, $\beta > 0$, and matrices $P$ and $K_i$ are solutions to the inequalities

$$
\begin{bmatrix}
\begin{bmatrix} P(A + \Delta A) + (A + \Delta A)^T P + \beta P + \beta^{-1} \Gamma(R) + \beta^{-1} \Gamma(S) K_i \end{bmatrix} \\
K_i \end{bmatrix} (C_1 + D_2 K_d)^T < 0
$$

then, there exists a decentralized robust $H_\infty$ state feedback controller (5), such that the resulting closed-loop system is asymptotically stable with the given $H_\infty$ performance index $\gamma$. Furthermore the matrix $K_d$ is a decentralized $H_\infty$ state feedback gain matrix.

Proof Using the solutions to Eqns. (7)-(9), two matrices $M$ and $N$ are defined as follows:

$$
M = P \Delta A + P \Delta A^T + P \Delta B, \quad K_i \Delta B_i^T P + \beta P + \beta^{-1} \Gamma(R) + \beta^{-1} \Gamma(S) K_i
$$

From lemma 1, lemma 2 and lemma 3, we can obtain

$$
J = \begin{bmatrix} M & P(a \Gamma(S)) \\
N & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\
0 & 0 \end{bmatrix} < 0
$$

Define matrix

$$
\Theta = \begin{bmatrix}
\begin{bmatrix} \Theta_{11} & PB_1 \end{bmatrix} (C_1 + D_2 K_d)^T \\
B_1^T P & -\gamma \end{bmatrix} \\
C_1 + D_2 K_d & -\gamma
\end{bmatrix}
$$

where

$$
\Theta_{11} = P(A + \Delta A) + (A + \Delta A)^T P + P(B_1 + \Delta B_1) K_i (B_2 + \Delta B_2)^T P
$$

Leading to $\Theta = \Theta(P, K_d, \alpha, \beta) + J$, it is clear that $\Theta < 0$ from the fact that $\Theta(P, K_d, \alpha, \beta) < 0$ and $J \leq 0$. Based on the bounded real lemma[13] we can conclude that the closed-loop system is stable with the disturbance attenuation level $\gamma$.