Catastrophe model and its experimental verification of static loading rock system under impact load

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Abstract: According to the catastrophe model for impact buckling of static loading structures, a new catastrophe model for impact loading failure of a static loading rock system was established, and one dimension (1D) catastrophe model was analyzed. The analysis results indicate that the bifurcation collection where catastrophe may take place is not only decided by mechanical system itself but also relates to exterior loading, which is different from the results obtained under mono-static loading where the bifurcation collection is only determined by mechanics of the system itself and has nothing to do with exterior loading. In addition, the corresponding 1D coupled static-dynamic loading experiment is designed to verify the analysis results of catastrophe model. The test is done with Instron 1342 electro-servo controlled testing system, in which medium strain rate is caused by monotony rising dynamic load. The parameters are obtained combining theoretical model with experiment. The experimental and theoretical curves of critical dynamic load vs static load are rather coincided, thus the new model is proved to be correct.

Key words: static loading rock system; impact load; instability; catastrophic model; coupled static-dynamic loading

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1 INTRODUCTION

Failure and fragmentation of rock under coupled static-dynamic loading relates to a wide range of engineering domain, such as rock fragmentation, boring, explosive and rockburst. So a lot of investigations on instability and failure of rock undergoing coupled static-dynamic load have been done[1-12]. For example, the behavior of fragmentation and failure of rock under 1D and 2D coupled static-dynamic load has been investigated experimentally, the mechanism on damage, catastrophe, fracture, constitutive model and strength criterion of rock under coupled static-dynamic load has been studied theoretically. Therefore, using catastrophe theory to study the fracture and failure of rock is an effective method, however, it is difficult to deal with dynamic stability problem of rock under impact loading using mathematical method, so quasi-static approach is usually used[13]. Although TANG[14] studied the instability failure process of rock under static loading using quasi-static method based on catastrophe theory, and ZUO et al[15] studied the characteristics of statically loaded rock under dynamic disturbance using catastrophe theory, there was very few investigations on instability failure of rock under impact loading with catastrophe theory, and there was even fewer studies on instability failure of statically loaded rock under impact loading using catastrophe theory.

There is similarity to some extent between buckling of structure and fracture and instability of rock. To study buckling of structure, impact buckling of statically loaded structure was studied using typical mechanics[14,15]; Non-linear static and dynamic elastic buckling of simple imperfect twin-bar frames was analyzed with catastrophe theory[16], and the critical load of static and dynamic buckling of frames could be calculated, respectively. Catastrophe model on impact buckling of frame and catastrophe model on impact buckling of pre-statically loaded frame were established[17,18], respectively. The purpose of this paper is to establish a catastrophe model on instability failure of statically pre-loaded rock under impact loading on the basis of Refs.[17,18] and to design a related experiment to verify the theoretical results.

2 CATASTROPHE MODEL ON INSTABILITY FAILURE OF PRE-STATICALLY LOADED ROCK UNDER IMPACT LOADING

In order to analyze the failure process of loading system using catastrophe theory, the loading system consists of experimental machine and sam-
ple. The results indicate that the static critical load is a bifurcational set of catastrophe system, in which potential function is potential energy of statically loading system of rock\(^{[17,18]}\). When rock undergoes impact load, the critical load of instability failure of rock may be the bifurcational set of catastrophe system, in which potential function \(R(x)\)\(^{[17,18]}\) is expressed as:

\[
R(x) = \int_{x_0}^{x} V(x) \, dx
\]

where \(x\) is generalized displacement of rock sample; \(V(x)\) is potential energy of loading system of rock.

It is noticed that according to the deduction of formula (1)\(^{[17,18]}\), the corresponding load is not the real impact load, and monotony rising dynamic load that causes middle strain rate of medium can also satisfy formula (1).

Statically loaded rock system undergoes impact load, i.e., a sample first undergoes a static pre-load \(p\), then undergoes an impact load \(P\), and the sketch map of loading curve is shown in Fig. 1, where \(x_0\) denotes the stable balance position of sample under static preload \(p\) (\(p\) is apparently less than static compressive strength), \(V_p(x)\) is the potential energy of system corresponding to displacement \(x\) of sample under static preload \(p\). \(V_{p+p}(x)\) and \(V_{p+p}(x_0)\) are the potential energy of system corresponding to displacement \(x\) and \(x_0\) of loading system under load \((p+P)\), respectively. According to Ref. [18], the catastrophe model pre-static on instability failure of loading rock system under impact load can be expressed as: the critical load \(P\) of impact instability failure of rock loading system under static preload \(p\) is the bifurcational set of catastrophe system, in which potential function is \(R(x)\):

\[
R(x) = \int_{x_0}^{x} [V_{p+p}(x) - V_{p+p}(x_0)] \, dx
\]

where

\[
V_{p+p}(x) = \int_{0}^{x} Q(x) \, dx - (p + P)x
\]

(3)

\[
V_{p+p}(x_0) = \int_{0}^{x_0} Q(x) \, dx - (p + P)x_0
\]

(4)

where \(x = x_0 + z; z\) is the displacement of sample under impact load; \(Q(x)\) is the change rule of displacement \(x\) of sample under exterior load \(Q\). Apparently, when \(p = 0\) and \(x_0 = 0\), formula (2) becomes formula (1).

3 CATASTROPHE THEORY ANALYSIS OF IMPACT INSTABILITY FAILURE OF 1D STATICALLY LOADED ROCK SYSTEM

Now impact instability failure of 1D statically loaded rock system can be analyzed using catastrophe model on impact instability failure of statically loaded rock system.

3.1 Catastrophe model

To the loading system that includes experimental machine and sample, the mechanical model is established\(^{[11]}\), as shown in Fig. 2. To the sample whose section area is \(A\), and length is \(L\), the relation between load \(Q\) and deformation \(x\) is expressed as

\[
Q = \lambda xe^{-x/u_0}
\]

(5)

where \(\lambda = EA/L\), is the initial stiffness of rock sample; \(E\) is elastic module; \(u_0\) is the value of deformation corresponded to peak load of load-displacement curve. The potential function of system is the total energy of system:

\[
V_Q = \int_{0}^{x} Q \, dx + \frac{1}{2} k (a - x)^2 = \lambda u_0 [u_0 - (u_0 + x)e^{-x/u_0}] + \frac{1}{2} k (a - x)^2
\]

(6)

where \(k\) is stiffness of experimental machine; \(a\) is total displacement of system.

If the static pre-load is \(p\), the displacement of sample is \(x_0\), there is

\[
p = \lambda x_0 e^{-x_0/u_0}
\]

(7)