Kinematics and dynamics analysis of a three-degree-of-freedom parallel manipulator

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Abstract: Kinematics and dynamics analyses were performed for a spatial 3-revolute joint-revolute joint-cylindric pair (3-RRC) parallel manipulator. This 3-RRC parallel manipulator is composed of a moving platform, a base platform, and three revolute joint-revolute joint-column pairs which connect the moving platform and the base platform. Firstly, kinematics analysis for 3-RRC parallel manipulator was conducted. Next, on the basis of Lagrange formula, a simply-structured dynamic model of 3-RRC parallel manipulator was derived. Finally, through a calculation example, the variation of motorial parameters of this 3-RRC parallel manipulator, equivalent moment of inertia, driving force/torque and energy consumption was discussed. The research findings have important significance for research and engineering projects such as analyzing dynamic features, mechanism optimization design and control of 3-RRC parallel manipulator.

Key words: kinematics; dynamics; parallel manipulator; Lagrange formula

1 Introduction

The conception of parallel mechanism can be traced back to 1895 when mathematician Cauchy studied a “octahedron connected by revolute joints”. Research of parallel mechanism started from 1938 when Pollard proposed to adopt parallel mechanism to spray paints on the automobiles. In 1949, Gough proposed to use a parallel mechanism called Stewart platform mechanism to inspect the tyres, which was the first true application of parallel mechanism. In 1965, senior engineer Steward from UK proposed the six degrees of freedom parallel mechanism used in flight simulator, and it was called Steward platform. In 1978, HUNT [1] proposed to adopt Stewart platform mechanism as manipulator mechanism, hence creating the new era of research and application of parallel manipulator mechanism. In the past thirty years, scholars in mechanism in various countries across the world have carried out research in parallel manipulators with less degree of freedom [2–9], and proposed some parallel manipulators with less degree of freedom and significant application value, such as the Delta parallel manipulator [10], 3-revolute pair-prismatic pair-spherical pair (3-RPS) parallel manipulator [11] and 3-revolute pair-revolute pair-revolute pair (3-RRR) parallel manipulator [12].

In Refs. [13–15], research over issues of position analysis, working space and kinematics of 3-RRC parallel manipulator is conducted. However, literatures involving dynamics analysis of 3-RRC parallel manipulator are relatively few with very inadequate knowledge of its dynamic properties. Based on the kinematics analysis on 3-RRC parallel manipulator, in this work, the kinematics of 3-RRC parallel manipulator, as well as its equivalent moment of inertia, driving force/torque is presented by establishing a dynamic model of 3-RRC parallel manipulator based on Lagrange formula.

2 Kinematics analysis of 3-RRC parallel manipulator

Mechanism of 3-DOF spatial parallel manipulator is illustrated in Fig. 1. Both the moving platform \( P_1P_2P_3 \) and the base platform \( B_1B_2B_3 \) are rectangles, and the moving platform and the base platform of the system are connected via three chains \( B_iC_iP_i \) \((i=1, 2, 3)\). Each chain is composed of two revolute pairs (R pair for short) \( B_i \) and \( C_i \), and a column pair (C pair for short) \( P_i \) \((i=1, 2, 3)\). The axes of three kinematic pairs in each branch \( B_iC_iP_i \) are parallel with each other, and they are also parallel to...
the base platform $B_1B_2B_3$. A fixed coordinate system $Oxyz$ is built. The original $O$ is located at the intersection between the normal of the revolute pair $B_1$ in the base platform $B_1B_2B_3$ and the extending line of $B_2B_3$ (i.e., point $O$ is located at the geometric center of the base platform), and the $z$ axis points upwards and is perpendicular to the base platform; the $x$ axis and the $y$ axis coincide with the normal of revolute pair $B_1$ and $B_2B_3$, as shown in Fig. 1.

The widths of the base platform and the moving platform in the given 3-RRC parallel manipulator are $l_1=2b_2$ and $l_2=2b_3$, respectively, and the lengths of the base platform and the moving platform are $2b_1$ and $2b_3$, respectively. The coordinates of points $B_1$, $B_2$ and $B_3$ in the coordinate system are $(b_1, 0, 0)^T$, $(0, b_2, 0)^T$ and $(0, b_3, 0)^T$, respectively. The mass of the moving platform is $m_0$; the center of mass $P$ is located at the geometric center of the moving platform, and its coordinates is $(x_0, y_0, z_0)^T$. Both $B_iC_i$ and $C_iP_i$ $(i=1, 2, 3)$ are homogenous links, the masses of which are $m_{B_i}$ and $m_{C_i}$, respectively. The coordinates of the center of mass of $B_iC_i$ and $C_iP_i$ are $(x_{C_i}, y_{C_i}, z_{C_i})^T$ and $(x_{P_i}, y_{P_i}, z_{P_i})^T$, respectively. The lengths of $B_iC_i$ and $C_iP_i$ are $l_i$ and $l_{C_i}$, respectively, and the rotation angles of $B_iC_i$ and $C_iP_i$ are denoted by $\alpha_i$ and $\theta_i$, respectively, as shown in Fig. 1. Thus, from the independent vector loop equation of 3-RRC parallel mechanism, we can get the following equations:

$$\begin{align*}
1_1 \sin \theta_1 + l_{12} \sin \alpha_1 &= l_{21} \sin \theta_2 + l_{12} \sin \alpha_2 \\
1_1 \cos \theta_1 + l_{12} \cos \alpha_2 + l_1 &= l_{31} \cos \theta_3 + l_{12} \cos \alpha_3 + l_2 \\
l_{21} \sin \theta_2 + l_{22} \sin \alpha_2 &= l_{31} \sin \theta_3 + l_{12} \sin \alpha_3
\end{align*}
\tag{1}
$$

Take the derivative of Eq. (1) with respect to variables $\theta_1$, $\theta_2$, and $\theta_3$ (variables $\alpha_1$, $\alpha_2$ and $\alpha_3$ are the functions of variables $\theta_1$, $\theta_2$ and $\theta_3$, respectively. By simultaneous solution, we obtain

$$\begin{align*}
\frac{\partial \alpha_1}{\partial \theta_1} &= \frac{l_{12} \cos \alpha_1}{l_{12} \cos \alpha_2} \\
\frac{\partial \alpha_1}{\partial \theta_2} &= \frac{l_{12} \cos \alpha_3 \sin(\theta_2 - \alpha_2)}{l_{12} \cos \alpha_1 \sin(\alpha_3 - \alpha_2)} \\
\frac{\partial \alpha_1}{\partial \theta_3} &= \frac{l_{12} \cos \alpha_2 \sin(\alpha_3 - \theta_2)}{l_{12} \cos \alpha_1 \sin(\alpha_3 - \alpha_2)} \\
\frac{\partial \alpha_2}{\partial \theta_1} &= 0 \\
\frac{\partial \alpha_2}{\partial \theta_2} &= \frac{l_{12} \sin(\theta_2 - \alpha_2)}{l_{12} \sin(\alpha_3 - \alpha_2)} \\
\frac{\partial \alpha_2}{\partial \theta_3} &= \frac{l_{12} \sin(\alpha_3 - \theta_2)}{l_{12} \sin(\alpha_3 - \alpha_2)} \\
\frac{\partial \alpha_3}{\partial \theta_2} &= 0 \\
\frac{\partial \alpha_3}{\partial \theta_3} &= \frac{l_{12} \sin(\alpha_3 - \theta_2)}{l_{12} \sin(\alpha_3 - \alpha_2)}
\end{align*}
\tag{2}
$$

It is assumed that all links are homogenous links. Hence, in coordinate system $Oxyz$, the coordinates of the center of mass of $B_iC_i$ and $C_iP_i$ $(i=1, 2, 3)$, and those of center of mass $P$ of the moving platform can be expressed respectively as (coordinate value not given is zero):

$$\begin{align*}
B_iC_i: \begin{cases}
x_{C_i} = b_1 + \frac{1}{2} l_{11} \cos \theta_1 \\
z_{C_i} = \frac{1}{2} l_{11} \sin \theta_1
\end{cases}
\tag{5}
$$

$$\begin{align*}
C_iP_i: \begin{cases}
x_{P_i} = b_1 + l_{11} \cos \theta_1 + \frac{1}{2} l_{12} \cos \alpha_1 \\
z_{P_i} = l_{11} \sin \theta_1 + \frac{1}{2} l_{12} \sin \alpha_1
\end{cases}
\tag{6}
$$

$$\begin{align*}
B_2C_2: \begin{cases}
y_{C_2} = b_2 + \frac{1}{2} l_{21} \cos \theta_2 \\
z_{C_2} = \frac{1}{2} l_{21} \sin \theta_2
\end{cases}
\tag{7}
$$

$$\begin{align*}
C_2P_2: \begin{cases}
y_{P_2} = b_2 + l_{21} \cos \theta_2 + \frac{1}{2} l_{22} \cos \alpha_2 \\
z_{P_2} = l_{21} \sin \theta_2 + \frac{1}{2} l_{22} \sin \alpha_2
\end{cases}
\tag{8}
$$

$$\begin{align*}
B_3C_3: \begin{cases}
y_{C_3} = b_2 - l_3 + \frac{1}{2} l_{31} \cos \theta_3 \\
z_{C_3} = \frac{1}{2} l_{31} \sin \theta_3
\end{cases}
\tag{9}
$$

![Fig. 1 Schematic representation of 3-RRC parallel manipulator](image)