Grouting diffusion of chemical fluid flow in soil with fractal characteristics

ZHOU Zi-long(周子龙)1, DU Xue-ming(杜雪明)1, CHEN Zhao(陈钊)2, ZHAO Yun-long(赵云龙)1
1. School of Resources and Safety Engineering, Central South University, Changsha 410083, China; 2. Guangxi Double Elephant Construction Limited Liability Company, Nanning 530029, China

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Abstract: The chemical fluid property and the capillary structure of soil are important factors that affect grouting diffusion. Ignoring either factor will produce large errors in understanding the inherent laws of the diffusion process. Based on fractal geometry and the constitutive equation of Herschel-Bulkley fluid, an analytical model for Herschel-Bulkley fluid flowing in a porous geo-material with fractal characteristics is derived. The proposed model provides a theoretical basis for grouting design and helps to understand the chemical fluid flow in soil in real environments. The results indicate that the predictions from the proposed model show good consistency with the literature data and application results. Grouting pressure decreases with increasing diffusion distance. Under the condition that the chemical fluid flows the same distance, the grouting pressure undergoes almost no change at first and then decreases nonlinearly with increasing tortuosity dimension. With increasing rheological index, the pressure difference first decreases linearly, then presents a trend of nonlinear decrease, and then decreases linearly again. The pressure difference gradually increases with increasing viscosity and yield stress of the chemical fluid. The decreasing trend of the grouting pressure difference is non-linear and rapid for porosity ϕ>0.4, while there is a linear and slow decrease in pressure difference for high porosity.

Key words: grouting; diffusion; Herschel-Bulkley fluid; porous media; fractal; grouting pressure

1 Introduction

Grouting is a proven but complex method for sealing and stabilizing soils in various engineering applications. The effect of grouting diffusion depends not only on the characteristics and types of fluid material but also on the pore structure of soils.

At present, most researches on grouting diffusion concern the behavior of grouting fluid. For types of typical fluid models, the Newton model [1, 2], the power law model [3, 4], the Bingham model [5–8] and the Herschel-Bulkley (H-B) model [9], have been used in existing studies. Among them, the H-B model is an improved model that considers the cohesion force and rheological properties of fluid. With a rheological index n=1, the H-B model is reduced to the Bingham model. With yield stress τ0=0, the H-B model can be reduced to the power-law model, and it is reduced to the Newton model when n=1 and n=0. Thus, the H-B model is most appropriate and most frequently used for describing the fluid behavior of grouting fluid.

However, few studies have considered the effect of the micro-structure of soil in grouting. In fact, geomaterials, such as soil and rock have large amounts of void space and fissures, which form a pore structure that influences fluid diffusion. Studies of rock and soil mechanics indicate that the pore structure of soil and rock has fractal characteristics [10–12]. Fractal geometry can be used to describe and interpret these types of pore structure and their permeability [13, 14]. For example, TANG et al [15] studied the structure of porous metals based on fractal theory and deduced the relationships between the fractal dimension and the porosity. ALAIMO and ZINGALES [16] deduced the fractional-order transport equation for the laminar flow through fractal porous materials and studied the transport law of a viscous fluid. SEDEH and KHODADADI [17] investigated the effects of fluid flow in porous media with two fractal dimensions and deduced the analytical equation of water pressure in a fractal reservoir. However, little is known regarding the grouting diffusion law of fluid flow in porous media based on the fractal geometry model.

Based on fractal geometry and the constitutive equation for H-B fluid, a fractal model for H-B fluid flowing in a porous geomaterial is derived that accounts for the capillary parameters of the soil medium, the rheological characteristics of chemical fluid and the grouting parameters. The proposed model provides a...
theoretical basis for grouting design and helps to understand the chemical fluid flow in soil in engineering applications.

2 Fractal characteristics of porous media

A porous medium consists of a large number of capillaries, which are a set of voids or gaps separated by a solid skeleton, whose cumulative size and tortuosity distributions have been proven to follow the fractal scaling law [18–22]. The scaling relationship between the number of cumulative capillaries and diameter is given by the following equation:

\[ N(\geq \lambda) = \left( \frac{\lambda_{\text{max}}}{\lambda} \right)^{D_f} \]  

(1)

where \( \lambda \) is the diameter of a random capillary and \( \lambda_{\text{max}} \) is the maximum diameter. \( D_f \) is the fractal dimension of the capillary size, or the size dimension, with \( 0 < D_f < 2 \) in two dimensions and \( 0 < D_f < 3 \) in three dimensions. The size dimension of the capillary is characterized by the uniformity of pore structure. The smaller the fractal dimension, the more uniform the pore structure. \( N \) is the cumulative number of capillaries whose diameter is not less than \( \lambda \).

Differentiating on both sides of Eq. (1) with respect to \( \lambda \) results in the population of capillaries,

\[ -dN(\lambda) = D_f \lambda_{\text{max}}^{D_f-1} \lambda^{-D_f-1} d\lambda \]  

(2)

A porous medium consists of a large number of tortuous capillaries. The lengths of capillaries have been proven to follow the fractal scaling law [23–25],

\[ L = \lambda^{1-D_T} L_0^{D_T} \]  

(3)

where \( L \) and \( L_0 \) are the actual lengths and characteristic lengths of the capillary, respectively, and \( D_T \) is the fractal dimension of capillary tortuosity, or tortuosity dimension, with \( 1 < D_T < 2 \) in two dimensions. The tortuosity dimension of the capillary is characterized by the tortuosity degree of capillaries. The smaller the tortuosity fractal dimension, the closer the capillary to a straight line.

Differentiating on both sides of Eq. (3) with respect to \( L_0 \) results in the actual lengths of capillaries, then

\[ dL = \lambda^{1-D_T} L_0^{D_T-1} D_T dL_0 \]  

(4)

3 Grouting diffusion of Herschel-Bulkley fluid in fractal media

Herschel-Bulkley fluid diffuses into the porous medium under the action of the grouting pressure (Fig. 1), taking the horizontal direction as the flow direction of the chemical fluid. Here, the chemical fluid is assumed to be incompressible and homogeneous. The density of the chemical fluid is constant, and the gravity of the chemical fluid is negligible.

A Herschel-Bulkley fluid is a non-Newtonian fluid with yield or shear stresses, and its constitutive equation has the form [26],

\[ \tau_w = \tau_0 + \mu \gamma^n \]  

(5)

where \( \tau_w \) is the shear stress; \( \tau_0 \) is the yield stress; \( \mu \) is the viscosity coefficient; \( \gamma \) is the shear rate of fluids; \( n \) is the rheological index. As described before, when the initial yield stress is zero, the Herschel-Bulkley fluid can be reduced to the power-law fluid. When the rheological index of the fluids is one, the Herschel-Bulkley fluid acts like a Bingham fluid. The Herschel-Bulkley fluid can be reduced to the Newton model when \( n=0 \) and \( \tau_0=0 \). These relationships are shown in Fig. 2.

For a Herschel–Bulkley fluid, the volumetric flowrate through a single tortuous capillary can be predicted by [27,28]

\[ q(\lambda) = \frac{\pi \lambda^3 n}{8 \tau_w^2} \left( \frac{\tau_w - \tau_0}{\mu} \right)^{1/n} \left( \frac{\tau_w - \tau_0}{\mu} \right)^2 + \frac{2 \tau_0 (\tau_w - \tau_0)}{1 + 2n} \left( \frac{\tau_w - \tau_0}{\mu} \right)^{2/n} \]  

(6)

Combining Eqs. (5) with (6), we have:

![Fig. 1 Schematic of fluid diffusion in grouting: (a) Pore structure of soil; (b) Flow path of capillary](image)