Large Deflection Dynamic Response Analysis of Flexible Hull Beams by the Multibody System Method

ZHANG Xiaojun*, and WU Guorong

School of Naval Architecture and Civil Engineering, Zhejiang Ocean University, Zhoushan 316004, P. R. China

(Received November 14, 2006; accepted March 22, 2007)

Abstract In this paper the large deflection dynamic problems of Euler beams are investigated. The vibration control equations are derived based on the multibody system method. A numerical procedure for solving the resulting differential algebraic equations is presented on the basis of the Newmark direct integration method combined with the Newton-Raphson iterative method. The sub beams are treated as small deformation in the convected coordinate systems, which can greatly simplify the deformation description. The rigid motions of the sub beams are taken into account through the motions of the convected coordinate systems. Numerical examples are carried out, where results show the effectiveness of the proposed method.

Key words hull flexible beam; large deflection; multibody system method; nonlinear

DOI 10.1007/s11802-007-0205-4

1 Introduction

In the past two decades, with the development of modern systems towards higher speed and exactness and light weight, the appearance of more and more large flexible structural components and the extensive application of composite materials which will not exceed the elastic limit in the case of large deformation, geometrically nonlinear problems have become a very important research subject in many fields, including naval architecture, and a great deal of research work has been done. Belytschko et al. (2000) has published a comprehensive introduction to nonlinear finite element methods for solving large deformation problems. A total Lagrangian finite element formulation was developed by Pai et al. (2000) for general beams undergoing large displacements and rotations based on Jaumann stress and strain measures, an exact coordinate transformation, and a new concept of orthogonal virtual rotations. Zupan and Seije (2003) presented a new finite element formulation of the 'geometrically exact finite-strain beam theory'. Li (2003) compared the linear and approximate non-linear as well as finite deformation theory of beam, plate and shell structures. Lin (2000) derived the equilibrium equation and boundary condition of the large deflection flexible bar by variational method. The exact relations between strains and displacements for large deformation of elastic thin shells are given by Huang et al. (2002) based on rational simplification when the deflection is of the same order as the thickness of the shell. In this paper, the large deflection dynamic problems of Euler beams are studied. The dynamic control equations are derived by using the multibody system method (Leung et al., 2004; Wu and Zhong, 2005; Shabana, 1989). The applicability and accuracy of this method are demonstrated by the simulation results.

2 The Multibody System Method

2.1 Coordinate System

Consider an Euler beam undergoing large deformation. We divide the beam into several subbeams and establish a set of coordinate systems shown in Fig.1. The OXY coordinate system is a global coordinate system fixed in time and forms a single standard and as such serves to define the connectivity between different subbeams. For an arbitrary subbeam i (i=1,2,...,N), we select a body coordinate system OXY with axis X being tangential to the neutral surface of the subbeam. The global position vector of an arbitrary point P on subbeam i can be written as:

\[ r_P' = R + A \begin{bmatrix} x' \\ 0 \\ y' \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \] (1)

where \( R \) is the vector of the body reference origin in the global coordinate system, \( x' \) is the local position of \( P' \) in the coordinate system \( OXY \), \( y' \) is the deflection of the subbeam observed in the body coordinate system and \( A \) is the transformation matrix defined as:

\[ A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}. \] (2)
2.2 Kinetic Energy and Strain Energy

Assuming that the length $l_i$ of subbeam $i$ is small enough so that its deformation, in the coordinate system $O'X'Y'$, can be approximately expressed as

$$v_i(x', t) = \sum_{n=1}^{N_i} \phi_n(x') h_{i,n}(t) = N_i^i q_f^i,$$

(3)

where $N_i$ is the shape function matrix, which is composed of the bending vibration modes $\phi_n(x)$, $q_f^i$ is the vector of elastic coordinates which contains the time-dependent coefficients $h_{i,n}(t)$.

The kinetic energy expression can be written as

$$T^i = \frac{1}{2} \int_{V^i} \rho q_f^i T \rho q_f^i dV^i$$

$$= \frac{1}{2} \hat{q}_f^i^T \left[ I_{2x2} B^i B^i^T A^i N_i^i \right] \hat{q}_f^i,$$

(4)

where

$$\hat{q}_f^i = \left[ R^T \theta^i q_f^i \right]^T,$$

(5)

$$M^i = \int_{V^i} \rho \left[ I_{2x2} B^i B^i^T A^i N_i^i \right] dV,$$

(6)

where $\rho$ is the material density, and matrix $B^i$ is defined as:

$$B^i = \frac{\partial A^i}{\partial \theta} \left[ \begin{array}{c} x_i \\ N_i^i q_f^i \end{array} \right].$$

The strain energy expression of subbeam $i$ can be written as

$$U^i = \frac{1}{2} q_f^i^T \left[ \int_0^{l_i} EI \left( \frac{\partial^2 N_i^i}{\partial x^2} \right)^T \left( \frac{\partial^2 N_i^i}{\partial x^2} \right) dx \right] q_f^i,$$

(7)

where $E$ is Young’s modulus, $I$ is the inertia moment of the cross section.

2.3 Dynamic Equations

Define the generalized coordinates as:

$$q = \left[ \begin{array}{c} q^T \\ q^{2T} \\ \ldots \\ q^{N_i^i T} \end{array} \right]^T,$$

(8)

where $N_i$ is the total number of the subbeams. The total kinetic energy and strain energy of the beam can be written as

$$T = \sum_{i=1}^{N} T^i = \frac{1}{2} q^T M q$$

and

$$U = \sum_{i=1}^{N} U^i = \frac{1}{2} q^T K q.$$

Respectively the constraint conditions between the subbeams can be written as:

$$\theta^{i+1} - \theta^i = \frac{\partial \theta^{i+1}}{\partial x_i} = \frac{\partial N_i^i}{\partial x_i} . q_f^i,$$

(9)

Eq. (9) can be further written in the form

$$C(q, t) = 0.$$

(10)

Utilizing the Lagrange multiplier method, we obtain the system dynamic equations as

$$M \ddot{q} + K q - C_q^T \lambda = F + Q_e,$$

(11)

where $C_q^T$ is the constraint Jacobian matrix, $\lambda$ is the vector of Lagrange multipliers, $F$ is the generalized external force vector, and $Q_e$ are the total inertial forces due to the quadratic velocity terms, which are of the following form:

$$Q_e = -M \dot{q} + \frac{\partial}{\partial q} (q^T M \dot{q}).$$

Introducing energy dissipation by viscous damping, Eq. (11) can be written as

$$M \ddot{q} + D \dot{q} + K q - C_q^T \lambda = F + Q_e,$$

(12)

in which $D$ is the damping matrix. From Eq.(10) and Eq. (12), we obtain

$$\begin{bmatrix} M & -C_q^T \\ C_q & 0 \end{bmatrix} \begin{bmatrix} \dot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} F + Q_e - D \dot{q} - K q \\ Q_e \end{bmatrix},$$

(13)

where

$$Q_e = -C_q \dot{q} - 2C_q q - (C_q q) q$$

2.4 Numerical Procedures

Newmark direct integration algorithm and a modified