An improved recursive decomposition algorithm for reliability evaluation of lifeline networks

Liu Wei¹,²† and Li Jie¹,²‡

¹. Department of Building Engineering, Tongji University, Shanghai 200092, China
². State Key Laboratory for Disaster Reduction in Civil Engineering, Tongji University, Shanghai 200092, China

Abstract: The seismic reliability evaluation of lifeline networks has received considerable attention and been widely studied. In this paper, on the basis of an original recursive decomposition algorithm, an improved analytical approach to evaluate the seismic reliability of large lifeline systems is presented. The proposed algorithm takes the shortest path from the source to the sink of a network as decomposition policy. Using the Boolean laws of set operation and the probabilistic operation principal, a recursive decomposition process is constructed in which the disjoint minimal path set and the disjoint minimal cut set are simultaneously enumerated. As the result, a probabilistic inequality can be used to provide results that satisfy a prescribed error bound. During the decomposition process, different from the original recursive decomposition algorithm which only removes edges to simplify the network, the proposed algorithm simplifies the network by merging nodes into sources and removing edges. As a result, the proposed algorithm can obtain simpler networks. Moreover, for a network owning s-independent components in its component set, two network reduction techniques are introduced to speed up the proposed algorithm. A series of case studies, including an actual water distribution network and a large urban gas system, are calculated using the proposed algorithm. The results indicate that the proposed algorithm provides a useful probabilistic analysis method for the seismic reliability evaluation of lifeline networks.

Keywords: lifeline system; network reliability; path-based recursive decomposition algorithm; disjoint minimal path; disjoint minimal cut; network reduction; reliability bound

1 Introduction

A main challenge in the reliability analysis of lifeline systems, such as water distribution and urban gas supply networks, is how to evaluate the node-pair reliabilities (Li, 2005). The methods proposed by in previous research can be divided into two major categories: the Monte Carlo simulation approach and the probabilistic analytical algorithm. For the Monte-Carlo simulation approach, the precision of the simulation results cannot be easily estimated (Li, 2005). Therefore, many efforts have been focused on exploring the probabilistic analytical algorithm, which can theoretically obtain accurate results.

Since network reliability can be calculated by adding the probability of all disjoint minimal paths, path-based algorithms have been widely investigated by many researchers. In 1973, Fratta and Montanari (1973) first adopted the sum of disjoint products (SDP) method to compute network reliability. In 1975, Aggarawal and Misra (1975) introduced Boolean algebra into the SDP method, which simplified the disjoint process of minimal paths. Then, Abrahman (1979) and Liao (1982a and b), presenting the so called "four sum of disjoint theorems" respectively, further developed the disjoint minimal path method and developed the path-based algorithms into a classical method. Later, in order to improve efficiency and apply the method to complex networks, and combine it with series-edge and parallel-edge reduction technologies, Page and Perry (1988; 1989) adopted the factoring theorem to reduce the complexity of network reliability analysis. However, the method cannot simplify a network into a simplest network whose reliability can be calculated directly. Thus, its application is limited. A common characteristic of the above methods is that all disjoint products have to be enumerated to provide an accurate result. However, as the problem to search for all network paths is a non-polynomial hard (NP hard) problem, i.e., the number of the paths in a network can be expressed as an exponential function instead of a polynomial function of the number of the network edges or nodes, the above approaches are not available for use with large lifeline networks. For large lifeline networks, an approximate algorithm should be developed to evaluate the reliability which can satisfy a prescribed error bound. Dotson
and Gobien (1979) first proposed a real-time disjoint of minimal paths in a network and gave an example to calculate the approximate value of network reliability. Later in 1988, by replacing the edge-incidence matrix with an adjacency matrix and introducing the breadth-first search technology, a modified Dotson algorithm was developed by Yoo and Deo (1988). Also, Yoo and Deo (1988) compared four current algorithms and concluded that Dotson’s algorithm was the most efficient. Introducing the concept of "system structure function" and combining Dotson’s algorithm with computer storage, Li and He (2002) presented a recursive decomposition algorithm, in which the probabilistic inequality is used to control the computation time and ensure the error bound remained within the predefined scope. This algorithm provides an efficient approach for large network reliability evaluation. Despite the detailed descriptive algorithm, rigorous mathematical proof of the method has yet to be achieved.

In this paper, using the concept of system structure function and minimal path set, three theorems are introduced and proved. Based on these theorems and the original recursive decomposition algorithm, an improved algorithm is derived to calculate the network reliability. The proposed algorithm takes the shortest path from the source to the sink of a network as decomposition policy. Using the Boolean laws of set operation and the probabilistic operation principal, a recursive decomposition process is constructed. During the decomposition process, different from the original recursive decomposition algorithm which only removes edges to simplify the network, the proposed algorithm simplifies the network by merging nodes into sources and removing edges. As a result, the proposed algorithm can obtain simpler networks. Also, when decomposing the network, the disjoint minimal path set and the disjoint minimal cut set are simultaneously enumerated. Consequently, the probabilistic inequality can be used to yield results that satisfy a prescribed error bound. Moreover, for a network owning s-independent components in its component set, two network reduction techniques are introduced to speed up the proposed algorithm. The proposed algorithm has been applied to a series of case studies. The results show that it provides a useful probabilistic analysis tool for the seismic reliability evaluation of lifeline networks.

2 Structure function and related theorems

A lifeline network is a graph with a weight, defined as the success probability (reliability) subject to seismic wave excitation, assigned to each edge or node. In general, network-assigned weight can be classified into three categories: (1) edge weighted network, where only edges are assigned weights; (2) node weighted network, where only nodes are assigned weights; and (3) general weighted network, where both nodes and edges are assigned weights. In this paper, only an edge weighted network is considered. However, for the node weighted network and general weighted network, the proposed algorithm can also be used after some small changes are made. A detailed process was illustrated by Li and He (2002).

For an edge weighted network, each edge can be in one of two states, operative or failed. Therefore, the network also has two states, operative or failed. In order to use Boolean algebra operators, the operative state and failed state are represented by 1 and 0, respectively. Two terminal nodes are denoted as source and sink. A network structure function is represented by $\Phi(G)$ and defined as follows

$$\Phi(G) = \begin{cases} 1 & \text{if network operates} \\ 0 & \text{if network fails} \end{cases} \quad (1)$$

Apparently, if all edges of any path of a network are in an operative state, the network operates. Assuming the network structure function and all the edges in a network are Boolean variants, the network structure function can be written as

$$\Phi(G) = \bigcup_{j=1}^{l} A_{pl} \quad (2)$$

where $l$ is the number of paths of $G$, $A_{pk}$ is the $k$th path of $G$ and can be written as

$$A_{pk} = a_{i1}a_{i2}\cdots a_{ls} \quad (3)$$

where $a_{is}$ is an edge of $G$ and $s$ is the number of edges in $A_{pk}$.

For any path $A_{pk}$ in Eq. (3), if it is not a minimal path (MP), there must be a corresponding MP $A_{pl}$, which is formed by removing several edges from $A_{pk}$. According to Boolean calculation, $A_{pl} \cap A_{pl} = A_{pl}$ exists. As a result, the network structure function $\Phi(G)$ can be rewritten as

$$\Phi(G) = \bigcup_{i=1}^{m} A_{pi} \quad (4)$$

where $m$ is the number of MPs of $G$ and $A_{pi}$ is the $i$th MP of $G$.

For example, Fig. 1 is a bridge network with source $s$ and sink $t$ and $A_{pl}=acd$ is a path but not a MP. Here $A_{pl}=ad$ is its corresponding MP after removing

![Fig. 1 A bridge network](image-url)