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REFINED ELLIPTIC TROPICAL ENUMERATIVE INVARIANTS

BY

FRANZISKA SCHROETER

Fachbereich Mathematik (AD), Universität Hamburg
Bundesstrasse 55, 20146 Hamburg, Germany
e-mail: franziska.schroeter@uni-hamburg.de

AND

EUGENII ShUSTIN

School of Mathematical Sciences
Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University
Ramat Aviv, 69978 Tel Aviv, Israel
e-mail: shustin@post.tau.ac.il

ABSTRACT

We suggest a new refined (i.e., depending on a parameter) tropical enumerative invariant of toric surfaces. This is the first known enumerative invariant that counts tropical curves of positive genus with marked vertices. Our invariant extends the refined rational broccoli invariant invented by L. Göttsche and the first author, though there is a serious difference between the invariants: our elliptic invariant counts weights assigned partly to individual tropical curves and partly to collections of tropical curves, and our invariant is not always multiplicative over the vertices of the counted tropical curves as was the case for other known tropical enumerative invariants of toric surfaces. As a consequence we define elliptic broccoli curves and elliptic broccoli invariants as well as elliptic tropical descendant invariants for any toric surface.

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1. Introduction

Refined (i.e., depending on a formal parameter $y$) tropical enumerative invariants were introduced by F. Block and L. Göttsc...