“Open learning? Computeralgebra?... No time left for that...”

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**Abstract:** Nowadays mathematics teachers have to deal with two challenges concerning their classroom-arrangements: include new teaching methods and integrate computers. The title expresses the fear of many teachers when following these trends, that realizing both makes curricular prescriptions even more difficult to achieve. In contrast to this other teachers perceive these trends not as an impediment, but as a special opportunity to achieve aims in terms of contents and processes. It was intended to investigate the question whether impediment or opportunity by a research project at the University of Duisburg-Essen. Teaching material was developed to introduce investigating polynomial functions in an open classroom-arrangement integrating CAS.

According to the multi-faceted arrangement a complementary research design was chosen which collects qualitative and quantitative data. The qualitative part is an interpretive study based on video tapes. The quantitative part is an experimental large-scale study. The material was used in 45 classes (about 1200 students) from different schools in order to check if general conclusions can be drawn. The large-scale study also includes a post-survey and a comparative post-test. To understand the aims of the project it is necessary to grasp the idea of the material. Therefore chapter 1 points out the main ideas of the material, chapter 2 explains the focus of the research project and in chapter 3 you will find first results.

**Kurzreferat:** Heutzutage müssen Lehrpersonen im Mathematikunterricht sich mit zwei Herausforderungen bei der Unterrichtsgestaltung auseinander setzen: das Einbeziehen neuer Unterrichtsmethoden und das Integrieren neuer Technologie. Der Titel drückt die Befürchtung vieler Lehrpersonen aus, dass der Unterrichtsstoff noch schwerer zu bewältigen ist, wenn man diesen Trends folgt. Im Gegensatz dazu steht die Erfahrung anderer Lehrpersonen, dass das Befolgen dieser Trends kein Hindernis sondern durchaus Chance sein kann, Inhalts- und prozessbezogene Ziele gleichermaßen zu erreichen. Die Frage ob Hindernis oder Chance sollte im Rahmen eines Forschungsprojektes an der Universität Duisburg-Essen untersucht werden und führte zur Entwicklung einer Lernwerkstatt zur Untersuchung ganzzrationaler Funktionen mit integriertem Einsatz von Computeralgebra (CAS).

Entsprechend der Vielschichtigkeit des Unterrichtsmaterials wurde auch das Forschungsdesign vielfältig gewählt – eine Mischung aus qualitativen und quantitativen Untersuchungen. Der qualitative Teil bestand aus interpretativen Studien auf der Basis von Videoaufzeichnungen. Der quantitative Teil mit einem Abschlussfragebogen und einem vergleichenden Abschussertest ist eine experimentelle Studie über den Einsatz des Materials in 45 Klassen (mit ca. 1200 Schüler/innen), um damit auch die Generalisierbarkeit zu untersuchen. Im Folgenden wird zunächst das Material vorgestellt (Kapitel 1), bevor das Forschungsdesign (Kapitel 2) und erste Ergebnisse (Kapitel 3) beschrieben werden.

**ZDM-Classification:** C70, D44, I44, U64, U74

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1. **The teaching material**

At first the material will be presented to give an impression of what it looks like. Afterwards some theoretical aspects will be discussed concerning the topic, the classroom-organisation and the use of technology.

1.1 **The material**

The material is meant to be used as an introduction into the aspects of the investigation of polynomial functions with aspects of differentiation (like slope, zeroes, extrema, inflection point). Students of 11th grade (about 17 years old) receive a work folder on paper (Barzel / Fröhlich/ Stechini-Carp 2003) with a set of worksheets (called “modules”) which they deal with independently in groups of 4-6 students for about 6 weeks. Supplementary material concerning individual stations is laid out in the classroom. The whole organisation of the teaching is like a work-place or a circle with different ways and possibilities of approaching the topic. This kind of classroom-arrangement is called a “Lernwerkstatt” (translated: learning workshop).

The only previous knowledge the students must have is the idea of derivation. It is possible to proceed through the learning workshop in different ways. Different ways of learning are also usually possible within the modules. There is no sequential arrangement of tasks in this case and the suggestions are given as mindmaps instead (an example of one module is given in fig. 1). At several points in the learning workshop, a comparison of the different types of representation of a function (graph, term, table, situation) is taken as the theme and the advantages and disadvantages are discussed. One example is shown in figure 1.

A variety of different types of tasks are involved in the material to evoke different kinds of student activities, for example:
- tasks which demand open ended approaches (Becker/Shimada 1997, Herget 2000),
- tasks which stimulate discussions between the students,
- tasks which initiate flexibility between the different representations in different modules to address different learning types (Herget/ Jahnke2001) (compare module E, fig.1),
- tasks which integrate students’ own experiences and experiments (Barzel 2002)

The following types of tasks should give an impression of this variety of the tasks:

**Giving functions with concrete analysis assignments:** This is performed for example in module “E” (compare fig. 1). Three different functions are given, one as a graph, one as a table and one as a formula. Without further previous knowledge concerning extreme points and their properties students have to recognise the properties by analysing the three examples. In module “L” (higher derivatives) graphs of a function and its derivatives are analysed and connections between the
grade of the polynomials and the maximal number of zeroes, extrema, points of inflection are recorded in structured form in a table.

Discussion of statements: If you concern yourself critically with a predetermined statement, you reflect on and interlink knowledge already acquired in order to arrive at an appropriate assessment. Therefore, assessment of the statement \( f'(x_0) = 0 \) \( \Rightarrow \) An extremum exists in \( x_0 \) results in development of an arithmetical method for determination of local extreme values (module “E” – see fig.1).

Text analysis: In module L (higher derivatives) information concerning higher derivatives is given; the module K (curvature) requires research on the topic of “point of inflection”. By means of structuring and separation of important and unimportant aspects, dealing intelligently with mathematical texts is practised.

To give teachers an idea of how to use the material in their classroom teaching, an introductory booklet serves as a guideline with main ideas and recommendations for realising the workshop in their own teaching. The booklet contains as well additional material for laying out in the classroom. One recommendation concerns the documentation of the learning process in a kind of journal, in which you cannot only find tasks and results but also ideas, individual examples, meanderings, highlights etc. (Hußmann 2003, Ruf/ Gallin 1998). In order to evaluate the learning process the teacher has several possibilities: s/he can assess the group work by judging the individual student's participation and engagement during the group work and the way of presenting results (by poster or other visualisation) and apart from that the teacher can check the journals of the individual student and of course the results of the final written test.

1.2. Theoretical aspects concerning the topic
As the teaching material was supposed to convince the “average” teacher to think about a change of his/her teaching, the creation of the teaching material was firstly a matter of clarifying the question of the topics based on which the example was to be set. The following reasons led to the choice of the topic “investigating a function”:
- It is a mandatory topic and not an additional teaching topic.
- It is a topic on higher secondary level, where teachers see usually a more appropriate place for computer algebra than in lower secondary level.
- The topic “investigating a function” is quite often taught as a fixed procedure which has to be done in certain pre-determined steps with drawing the graph of the function at the end. This scheme provides much opportunity for criticism and is above all perceived as unsatisfactory by the teachers themselves. The core of the criticism in addition to the lack of satisfaction lies in the fact that the underlying mathematics is not understood by the students. Instead they often blindly follow a certain scheme and use formulas.
- “Functions” as a mathematical topic is a wonderful example of showing the benefits of involving CAS into the learning and teaching process. CAS offers the

Experimental experience: “Derivative graph walking” is a module that encourages trials with a sonic motion detector (CBR – “Computer Based Ranger coming along with a TI-calculator). Movements are recorded indirectly as a time-distance or time-speed diagram. A graph of a derivative is produced in this manner by one’s own walking. This type of graph is analysed and conversely, predetermined graphs are followed and matched. Cognitive discussions are linked as a result with concrete experience, in order to facilitate comprehension of the new contents.

An overall reflection of the workshop is finally performed by preparing posters for a final presentation.

Fig 1: One module as an example: Module E - Extrema

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>13,5</td>
</tr>
<tr>
<td>1</td>
<td>5,94</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-1,69</td>
</tr>
<tr>
<td>4</td>
<td>-2,5</td>
</tr>
<tr>
<td>5</td>
<td>-1,81</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
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<td>12,81</td>
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<tr>
<td>12</td>
<td>13,5</td>
</tr>
<tr>
<td>13</td>
<td>12,69</td>
</tr>
</tbody>
</table>

Find a calculation to determine local extrema. Use this calculation for the functions given by the following equations. Check by plotting the graphs.

Function 1:
\[
f(x) = x^2 + 2\]

Function 2:
\[
f(x) = x^2 + 2\]

mit \(-2 \leq x \leq 2\)

Function 3:
\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
0 & 13,5 \\
1 & 5,94 \\
2 & 1 \\
3 & -1,69 \\
4 & -2,5 \\
5 & -1,81 \\
6 & 0 \\
7 & 2,56 \\
8 & 5,5 \\
9 & 8,44 \\
10 & 11 \\
11 & 12,81 \\
12 & 13,5 \\
13 & 12,69 \\
\hline
\end{array}
\]

If the first derivative is 0, then there is a minimum! – Discuss this statement and correct it if necessary.

You can see three functions in different representations. Determine the local extrema and try to define the concept “local extrema”. What are the benefits and problems of the different representations?