Opportunities for the Use of Computer Algebra Systems in Middle Secondary Mathematics in England and Wales

David Bowers, Ipswich

Abstract: This paper presents and discusses a number of ways in which mathematical attainment targets specified for pupils aged 11–16 in the revised National Curriculum can be achieved when teachers deploy computer algebra systems.


ZDM-Classification: D10, R20

1. Introduction

In England and Wales – the system is slightly different in other parts of the United Kingdom – schooling is compulsory to the age of 16. Thereafter, pupils may follow a two-year course of further academic study (A-level) in preparation for university, pursue vocationally-oriented education and training, or seek employment.

The way in which computer algebra systems (CAS) might influence the teaching, learning and assessment of post-16 mathematics, in particular at A-level, has been widely discussed, among others by the Association of Teachers of Mathematics (1995), Bowers (1995a), Hunter (1993), Taylor (1995) and Williamson (1992). Comparatively little has been written on the use of CAS in pre-16 mathematics. One reason for this is that those areas of mathematics which suddenly become immediately accessible using symbolic manipulation software, such as the routine performance of techniques in algebra and calculus, are not normally met in any great detail until A-level or equivalent courses.

This paper will focus on middle secondary education, and highlight those areas of mathematics where CAS might be deployed meaningfully. This will be strictly within the context of the National Curriculum. Reference will be made to published studies, and some novel examples will also be introduced and discussed.

2. The National Curriculum for mathematics in England and Wales

The revised National Curriculum (Department for Education 1995) applies to pupils of compulsory school age and is organised on the basis of four Key Stages (age 5–7, 7–11, 11–14 and 14–16 respectively). Thus KS3 and KS4 correspond to secondary education. Mathematics is compulsory at all Key Stages.

For each subject and at each Key Stage there are programmes of study which set out what pupils should be taught. In mathematics at KS3/4 the programme of study consists of five sections: Number; Algebra; Shape, Space and Measures; Handling Data; Using and Applying Mathematics. There is an additional section, Further Materials, for more able pupils at KS4.

The expected standards of pupils’ performance in each section of the programme of study are set out explicitly in attainment targets, described in eight levels of increasing difficulty. At KS3/4 it is expected that pupils will be able to record achievement in the attainment targets at levels 5 to 8. There is an additional level description above level 8 to help teachers in differentiating exceptional performance.

An example may help to clarify the above. The attainment targets for the programme area Shape, Space and Measures include “pupils use everyday language to describe properties and positions ... when working with 2-D and 3-D shapes” (level 1), “pupils identify all the symmetries of 2-D shapes” (level 5) and “pupils use sine, cosine and tangent in right-angled triangles when solving problems in two dimensions” (level 8). The reader will be correct in concluding that teachers in England and Wales have a considerable task to plan their teaching and assessment, and to record the results, to ensure that each pupil has the opportunity to demonstrate competence in the various skills at the appropriate level in each of the attainment targets for all of the sections of the programme of study.

At the end of their final year of compulsory schooling, at age 16, pupils take externally set and marked examinations (General Certificate of Secondary Education, or GCSE) in each subject, which provide a formal summative assessment of their attainment in the National Curriculum. In mathematics, the GCSE usually consists of two written examination papers and a piece of assessed coursework.

3. Scope for using computer algebra systems at Key Stages 3 and 4

Each section of the mathematics programme at KS3/4 will now be considered, and areas which are amenable to the use of CAS will be highlighted. Particular reference will be made to the computer programme Derive, since a recent survey (Bowers 1995b) indicates that this is by far the most common choice of software in those schools where CAS are available for staff or student use. However, the comments could be modified to suit other symbolic manipulators.

3.1 Number

This section of the National Curriculum is encouragingly prefaced with the requirement that “pupils should be given opportunities to use calculators and computer software, eg spreadsheets”. Indeed, simple scientific calculators will be found in every mathematics classroom, and it may be assumed that the majority of pupils will experience using a spreadsheet during their secondary education, not only in mathematics but also in other subjects. However, there are some aspects of Number for which a CAS is the most appropriate tool.

One advantage of a CAS is its ability to display large numbers in full, and decimal numbers to a high level of precision. This should allow greater confidence and understanding in the topic “decimals, ratios, fractions and percentages, and the inter-relationships between them”, especially with regard to the decimal representation of frac-
Decoding messages relies on the ability to factorise large numbers, a task which can now be delegated to the computer. Pupils are initially keen to decode messages such as

18366000961262524628803026963045247176133890

and send similar messages to their friends. More able pupils can be encouraged to consider the underlying mathematics. How could we tell in advance that the first word in the example above has just two letters? Does the second word contain an A? When pupils tire of typing in large numbers, the challenge can be set to devise a more efficient coding method (Bowers 1997).

3.2 Algebra

It has been pointed out in a variety of studies (for example Sutherland 1990, Sutherland and Pozzi 1995) that the amount of algebra taught pre-16 has diminished in recent years, and that it is possible to gain a reasonably good grade in the GCSE examination without mastering the algebra techniques specified in the higher levels of attainment. However, there is scope for using CAS to support most of the algebra topics in the National Curriculum, which may help to make them more accessible to pupils.

At KS3 (level 5), students are expected to “construct, express in symbolic form, and use simple formulae involving one or two operations”, having previously used “simple formulae expressed in words”. The word input mode of a CAS such as Derive can aid this shift to generality (see also Pozzi 1993). Thus the bill from a plumber who charges £12 per hour for labour plus a call-out fee of £20 could be expressed by

\[ \text{cost} = 20 + \text{hours} \times 12 \]

and entered as such into the CAS in word input mode. The cost of a job lasting 3 hours can be found by using the “substitute” command to replace hours by 3, and then using the “solve” command. This is a valuable check for the pupil, since the CAS will definitely perform the operations in the correct order! Also, when using a CAS it is no more difficult to substitute a figure for cost and solve to find the number of hours worked. Doing this by hand is deemed by the National Curriculum to be a higher level skill (level 6), yet within a CAS environment it would appear both easy and obvious that a functional relationship such as this can work both ways, paving the way for a more formal algebraic treatment of the solution of linear equations.

The use of Derive’s vector function

\[ \text{vector([hours, cost], hours, 1, 8)} \]

with the cost function declared as above, generates a table of values which can be plotted directly on suitable axes (Fig. 1). Admittedly the syntax of the vector function may not be immediately obvious to pupils at KS3/4, but it can be argued that this function is so powerful and versatile that it is worth spending time to become familiar with it (Bowers 1996). Although a spreadsheet may be able