A New Mean Reversion Model of Close-End Fund

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Abstract: On the basis of fractal theory, one of the nonlinear theories, this paper studies the validity of Chinese fund market fractal time sequence through Hurst exponent, calculates the $H$ value and proposes a new close-end fund mean reversion model. Meanwhile, this paper validates the mean reversion time sequence for consecutive 54 week data of fund market. The result indicates that this model can effectively prove that Chinese close-end fund market follows the biased random walk. The research also proves that the fund discount does have mean reversion tendency and averagely the fund with high discount has a higher excess yield than that of the fund with low discount. The mean excess yield and the ratio between discount rate deviation and standard deviation demonstrate a descending relationship. The optimum investment period based on “mean reversion” is one month. Consequently this model provides a new arbitrage method through the discount of close-end fund.

Key words: close-end fund; Hurst exponent; mean reversion model; arbitrage opportunity

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Introduction

The closed-end fund transaction\textsuperscript{[1]} is a normal phenomenon in both domestic and oversea fund market. In the past thirty years, experts and scholars spent much time and energy on this issue to find out an answer, and they proposed various theories to explain the riddle of closed-end fund discount. Most scholars explain the discount phenomena through noise theory and fractal theory\textsuperscript{[2,3]}. Nonetheless, these theories can not integrally explain the universal law of closed-end fund discount. There is only limited research in this area in China because of the short history of China’s close-end market. Thus the study for close-end fund discounts and arbitrage opportunity is important and realistic.

With the development of fractal and chaotic theory, the foreign scholars began to use dynamic nonlinear theory to validate the efficiency of capital market from 1980’. These positive researches bring more and more suspicion to the traditional effective market assumption\textsuperscript{[4]}. The research of chaos and fractal properties for security market has become a flourishing topic\textsuperscript{[5]}. Recently some Chinese scholars began positive analysis for China’s security market through Rescaled Range Analysis (R/S analysis). Nevertheless, the analysis mainly focuses on fractal properties of security market\textsuperscript{[6,7]}. There are still no articles on the study of Hurst Exponent through R/S analysis, as well as the study on mean reversion property of close-end fund.

The article calculates the Hurst Exponent of investment through R/S analysis establishes a new mean reversion model to evaluate security investment fund and accordingly proves that Chinese close-end fund has the
property of mean reversion. This research indicates the discount and arbitrage opportunity of Chinese close-end fund.

1 The H Value Calculation of Hurst Exponent

Hurst studied in all aspects on the biased random walk in 1940s. Mandelbrot did the detailed research again in 1960s and 1970s and called it fractal Brownian motion. Now it is called fractal time series[6]. Testing the fractal time series through Hurst exponent is the basis of validation for market efficiency. The key is to calculate the H value, H is Hurst exponent.

Consider a time series of even time intervals \( \{X(t_i)\}, i=1,2,\cdots,n \). For a random \( t_j \in \{t_i : i=1,2,\cdots,n\} \), its mean is defined as:

\[
\bar{X}(t_j) = \frac{1}{j} \sum_{i=1}^{j} X(t_i)
\]

where \( t \) is the running sample value.

Accumulated deviation:

\[
Y(t_i,t_j) = \sum_{t_i \leq t \leq t_j} (X(t) - \bar{X}(t_j)) , \quad i = 1,2,\cdots,j
\]

Range:

\[ R(t_j) = \max_{t_i \leq t \leq t_j} Y(t_i,t_j) - \min_{t_i \leq t \leq t_j} Y(t_i,t_j) \]

Standard deviation:

\[
S(t_j) = \sqrt{\frac{1}{j} \sum_{i=1}^{j} (X(t_i) - \bar{X}(t_j))^2}
\]

when \( \{X(t_i)\} \) is a random series of relative independency and limited variance, i.e. the Brownian motion, Hurst had proved the following result:

\[
R(t_j) / S(t_j) = (X(t_j)/2)^{H_0}
\]

where Hurst exponent \( H_0 = \frac{1}{2} \).

When \( \{X(t_i)\} \) is a fractal Brownian motion and is not relatively independent, in other words, the fractal dimensions time series, \( R(t_j) / S(t_j) \) follows[6]:

\[
R(t_j) / S(t_j) = C t_j^{H_0}
\]

where \( C \) is a constant. Logarithmize both sides of Eq.(1), we get:

\[
\ln R'(t_j) / S(t_j) = \ln C + H_0 \ln t_j
\]

Where \( R'(t_j) \) is rescaled range, \( C \) is a constant and \( t_j \) is the number of samples observed. Apply the least square method to Eq.(2) to regress the slope, now we get the Hurst exponent \( H_0 \). Then we can further work our the fractal dimensions \( D_0 \) of time series. Generally, the R/S analysis method is denoted as:

\[
(R'/S)_n = C' \times n^{H_0}
\]

where \( R' \) : rescaled range, \( C' \) is a constant, \( n \) is the number of samples observed.

Logarithmize both sides of Eq.(3), then

\[
\ln(R'/S)_n = \ln C' + H_0 \ln n
\]

Therefore we can work out a chart for \( \ln(R'/S)_n \) and \( \ln n \) and get Hurst exponent \( H_0 \).

There is a relation between Hurst Exponent and the value of fractal dimensions \( D_0 \) as follows:

\[
D_0 = 2 - H
\]

According to the relationship of \( \ln(R'/S)_n \) and \( \ln n \), when \( H > \frac{1}{2} \), \( 1 < D_0 < 1.5 \); When \( H < \frac{1}{2} \), \( 1.5 < D_0 < 2 \).

Because fractal dimensions \( D_0 \) reflects the non-smoothness and acuteness of the motion track, for fractal time series, its motion track will be less smooth and fluctuating more acutely as reducing \( H_0 \) and increasing \( D_0 \).

2 The Proposal of Mean Reversion Model and Validation

Hurst exponent only requests limited assumptions for the phenomena to be described and can be applied to many time series[10, 11]. Thus it is possible to set up a mean reversion model and use a time sequence to test the correctness of the mean reversion model and measure the intensity of the trend.

2.1 The Establishment of Mean Reversion

First define a difference concerning the weighted average discount price rate of closed-end fund:

\[
R_{ts} = \sum_{i=1}^{w} \left( \text{Disc}_{ts} - M_u \right)
\]

\( R_{ts} \) is accumulative deviations of \( n \) periods, \( \text{Disc}_{ts} \) is weighted average discount price rate of the week \( u \). \( M_u \) is mean of \( \text{Disc}_{ts} \) in \( n \) periods.

The difference is the margin between maximum value and minimum value in Eq.(5):

\[
R = \max(R_{ts}) - \min(R_{ts})
\]

In Basic Econometrics, we have proved that

\[
C(t) = 2^{(2H_0-1)} - 1
\]

where \( C(t) \) is correlation coefficient[3].

When \( 0 \leq H_0 < 0.5 \), \( C(t) < 0 \), the time series is negatively correlated. This kind of system is “anti-prolonged” or periodic time series, called “mean reversion”