Fractal Characteristics of Particle Size Distribution in Dynamic Flocculation Process

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Abstract: This study selected polyaluminum chloride (PAC) coagulant to remove suspended particles in Kaolin suspension solution and used a turbidimeter and particle counter to monitor the flocculation process online and collected the experiment data. The experiments were conducted to study the dynamic distribution characteristics of suspended particles under different hydrodynamic conditions. The results show the self-similarity and scale invariance of particle size distribution. The study further proposed the concept of fractal dimension of particle size distribution and found out that fractal dimension changed in a similar way as residual turbidity did and could excellently indicate the variation of coagulation effect. Therefore, fractal dimension could be adopted to optimize the addition of coagulants and the quality of outflow could be further improved to reduce production costs.

Key words: flocculation; particle size distribution; fractal dimension; particle counter

CLC number: TU 991

Introduction

Because flocculation is one of the key techniques generally adopted in water supply and treatment and has a major impact on subsequent operation procedures, quality of ultimate outflow and production costs, the process has become a significant domain for scientific researches in environment engineering. The process is often characterized by complexity and chaos due to the presence of floc particles that are loose, hard to sediment, or easy to break apart. Until today, the knowledge of floc formation, solution chemistry, hydrodynamics and interaction among the substances (i.e., various matters produced during the flocculation process) that constantly aggregate and separate is limited. The introduction of fractal theory made possible a better understanding of the disorder and irregularity of floc particles in the flocculation process. In the late 1980s, researchers worldwide introduced the concept of fractal characteristics based on the randomness and non-linear characteristics of the process and the results of related researches[1]. However, few reports introduced fractal characteristics into the study of particle size distribution. In recent years relevant studies have been focusing on solving the equations of coagulation dynamics[2]. Research was also conducted on particle population and particle size distribution in natural water to prove its fractal characteristics. Jin et al[3] studied raw humic acid with image method and proposed that its particle size distribution conformed to the law of normal distribution.

With a particle counter to effectively monitor the
dynamic particle distribution in water\cite{4}, the research explored the law of particle size distribution in the flocculation process using particle size distribution function and statistical methods. It discovered the fractal characteristics of particle size distribution and proposed the concept of fractal dimension. Through a series of tests, the impact of coagulant dosage and hydrodynamics was studied on fractal characteristics of particle size distribution in the flocculation process. The results show that there is an excellent correlation between fractal dimension and coagulation effect.

1 Fractal Dimension of Particle Size Distribution

The particle size distribution function generally takes the diameter variable (or surface area, volume etc.). The following (with diameter variable) is the function defined by Friedlander\cite{5}:

\[ n(d_p) = \frac{dN}{d(d_p)} \]  

where \( n(d_p) \) is the particle size distribution function; \( d_p \) is the diameter of floc particles; \( dN \) is particle population between \( d_p \) and \( d_p + d(d_p) \).

Based on the findings of Friedlander\cite{5} and Logan et al\cite{6}, the relationship between the concentration of particles in water (i.e., the particle size distribution function) \( n(d_p) \) and the particle diameter \( d_p \) can be described by the following power function:

\[ n(d_p) = k d_p^{-\beta} \]  

where \( k, \beta \) are invariables.

The following equation was returned when taking natural logarithm at both ends in formula (2):

\[ \ln[n(d_p)] = -\beta \ln d_p + \ln k \]  

Tests on the raw water from Songhua River, the effluent from the sedimentation tank and that from the filter tank of a water plant based on formula (3) proved that there was an approximate linear relationship between particle population per unit volume of water \( n(d_p) \) and particle dimension \( d_p \) was reflected in the dual-logarithm coordinates system in Fig. 1. As shown in Fig.1, between \( n(d_p) \) and \( d_p \) there is no linear relationship in the system. In other words, the formula (3) applies to the research on fractal characteristics of particle size distribution in the flocculation process.

Formula (2) was then modified as the following:

\[ N(r) = \int_0^r dN = \int_0^r n(d_p) d(d_p) \]

\[ = \int_0^r k d_p^{-\beta} d(d_p) = \frac{k}{1-\beta} r^{1-\beta} \]  

where \( r \) is the diameter of floc particles; \( N(r) \) is the particle population with diameter less than or equal to diameter \( r \).

\[ p(r) = \frac{N(r)}{N} \] was defined as the cumulative distribution function of particle size distribution in flocculation process:

\[ p(r) = \frac{N(r)}{N} = \frac{k}{N(1-\beta)} r^{1-\beta} \]  

where \( N \) is the total number of floc particles in water.

Formula (5) shows that there was a power exponent relationship between \( p(r) \) and \( r \). The following was returned when taking natural logarithm at both ends in formula (5):

\[ \ln[p(r)] = D_f \ln r + \ln K \]  

where \( D_f = 1 - \beta, K = \frac{k}{N(1-\beta)} \).

Formula (6) shows that there was a linear relationship between \( p(r) \) and corresponding upper limit diameter \( r \) in the dual-logarithm coordinates system. \( D_f \) was the straight slope, which was then defined as the fractal dimension of particle size distribution, or fractal dimension and was used as a parameter to reflect the variations of floc particles and the coagulation effect.

2 Materials and Methods

2.1 Instruments and Agents

Instruments used in the study included PCX2200