Existence and global attractivity of positive periodic solutions of competition systems

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Abstract In this paper, we study the existence and global attractivity of periodic solutions of a competition system. We obtain sufficient conditions for the existence and global attractivity of positive periodic solutions by Krasnoselskii’s fixed point theorem and the construction of Lyapunov functions.

Keywords Positive periodic solutions · Competition system · Lyapunov function · Krasnoselskii’s fixed point theorem

Mathematics Subject Classification (2000) 34C25 · 34D20

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1 Introduction

In this paper, we investigate the following delayed three species competition system

\[
\begin{align*}
    x_1'(t) &= x_1(t) \left[ r_1(t) - a_{11}(t)x_1(t) - a_{12}(t) \int_{-\infty}^{t} K_{12}(s-t)x_2(s)ds - a_{13}(t) \int_{-\infty}^{t} K_{13}(s-t)x_3(s)ds \right] \\
    x_2'(t) &= x_2(t) \left[ r_2(t) - a_{21}(t) \int_{-\infty}^{t} K_{21}(s-t)x_1(s)ds - a_{22}(t)x_2(t) - a_{23}(t) \int_{-\infty}^{t} K_{23}(s-t)x_3(s)ds \right] \\
    x_3'(t) &= x_3(t) \left[ r_3(t) - a_{31}(t) \int_{-\infty}^{t} K_{31}(s-t)x_1(s)ds - a_{32}(t) \int_{-\infty}^{t} K_{32}(s-t)x_2(s)ds - a_{33}(t)x_3(t) \right],
\end{align*}
\]

where \( x_i(t) \ (i = 1, 2, 3) \) denotes the density of competing species at time \( t \), \( r_i, a_{ij} \in C(R, [0, +\infty)) \) are \( \omega \)-periodic functions \((\omega > 0)\), \( K_{ij} \ (i = 1, 2, 3; \ j = 1, 2, 3) \) is a nonnegative function in \( L_1(-\infty, 0] \) with

\[
\tilde{r}_i = \frac{1}{\omega} \int_{0}^{\omega} r_i(s)ds > 0, \quad i = 1, 2, 3,
\]

\[
k_{ij} = \int_{-\infty}^{0} K_{ij}(\theta)d\theta > 0, \quad a_{ij}^K = \frac{1}{\omega} \int_{0}^{\omega} a_{ij}(\sigma)d\sigma \int_{-\infty}^{\sigma} K_{ij}(s-\sigma)ds > 0,
\]

In recent years, many authors have researched the theories of functional differential equations in mathematical ecology. The so-called mathematical ecology, which uses mathematical models to describe relations between the creatures and the environment, and uses mathematical methods (theoretical or computational) to study the ecological phenomenon in order to these phenomenon can be explained and controlled. Various mathematical models have been proposed in the study of population dynamics, ecology and epidemiology. One of the most celebrated model is the competition system. For instance, Kuang [1] proposed the following delayed three species competition system

\[
\begin{align*}
    x_1'(t) &= x_1(t) \left[ 1 - x_1(t) - \int_{-\infty}^{t} K(s-t)x_2(s)ds - \int_{-\infty}^{t} L(s-t)x_3(s)ds \right] \\
    x_2'(t) &= x_2(t) \left[ 1 - \int_{-\infty}^{t} L(s-t)x_1(s)ds - x_2(t) - \int_{-\infty}^{t} K(s-t)x_3(s)ds \right] \\
    x_3'(t) &= x_3(t) \left[ 1 - \int_{-\infty}^{t} K(s-t)x_1(s)ds - \int_{-\infty}^{t} L(s-t)x_2(s)ds - x_3(t) \right],
\end{align*}
\]