Reply to Discussion of “Application of Excel Solver for Parameter Estimation of the Nonlinear Muskingum Models” by Ali R. Vatankhah*

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Abstract

Flood routing is a mathematical procedure in order to predict the variation of the magnitude of a flood wave when it propagates along natural rivers or artificial channels. The accuracy of flood routing is an important subject for research in the hydrology and hydraulics. This study focuses on the effects of the numerical solution method and the nonlinear storage equation of the Muskingum flood routing procedure. The simulation results indicate that the Tung’s method has better results (up to 41.26-79.47% for the different nonlinear storage equations) than the Fourth-Order Runge-Kutta method. Therefore, the Tung’s method is more accurate than the other method. Furthermore, based on the previous studies, the Tung’s method has satisfactory results in the field conditions in both calibration and verification steps. On the other hand, the proposed five-parameter nonlinear storage equation improves fitting to the outflow data by 85.21% than the three-parameter equation.

1. Introduction

I would like to thank my colleague Dr. Ali R. Vatankhah, who provided the some comments regarding the nonlinear Muskingum models and their numerical solution procedure. However, the following responses are necessary to clear some obscure issues raised by the discusser.

The Muskingum procedure has three general components that are (1) a method for the calibration of hydrologic parameters (Luo and Xie, 2010; Barati, 2011; Xu et al., 2012; Orouji et al., 2013); (2) a storage-discharge relationship along with the one-dimensional continuity equation as governing equations (Chow, 1959; Gill, 1978; Easa, 2013); and (3) a method for the numerical solution of the governing equations (Tung, 1985; Easa, 2012). Each of these components has significant effects on the simulation of the Muskingum procedure.

As mentioned at the end of “Results and Discussion” section of the original paper, the main object of the original study was the evaluation of the Excel solver as efficient, simple and convenient tool for parameter calibration of the nonlinear Muskingum model through the benchmark problem. In other words, the approach in the original paper focuses on the performance of the calibration method of the hydrologic parameters of the Muskingum procedure. Therefore, the same storage-discharge relationship and numerical solution method that employed by other researchers should be used to make the results of different calibration methods comparable. Based on the results of the original paper, the Excel solver was a promising approach to reduce problems of the parameter calibration of the nonlinear Muskingum routing procedure.

In the present study, other issues of the Muskingum procedure will be considered. In section 2, some issues about the value of the time step are presented. In section 3, the general form of the storage equation of Muskingum procedure is derived, mathematically. Then, in section 4, the statistical criteria for evaluation of flood routing are discussed. Numerical solution methods of Muskingum flood routing procedure are compared in section 5. Finally, in section 6, conclusions are outlined.

2. Discussion of Time Step Value

The discusser mentioned that the value of K should be corrected because the time step value, Δt, was assumed to be unit time in the original paper and other previous studies (e.g., Tung, 1985; Luo and Xie, 2010; Xu et al., 2012; Easa, 2012; Orouji et al., 2013; Easa, 2013). However, Geem (2013) mathematically showed that there is no difference between the simulation results of the unit time step value and the real time step value using the discrete-time method of Tung (1985) if the value of K is consistent with time step value. In other words, two form of the calibrated parameter using either the unit time step or the real time step are true.

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3. Discussion of Storage Equation

The discusser combines two different forms of nonlinearity of Eqs. (8) and (9) of the original paper to improve fitting performance without any physically or mathematically justification. In this study, the general form of the storage equation of Muskingum procedure will be derived, theoretically.

As mentioned in the original paper, Chow (1959) assumed that the upstream and downstream end sections of the reach have the same mean discharge and storage relationships with respect to the flow depth. However, this assumption may not establish in all natural rivers. Therefore, the relationships of the mean discharge and storage of the upstream and downstream end sections can express as:

\[ I = A_1 y^{N_1}, Q = A_2 y^{N_2} \]  \hspace{1cm} (32)

\[ S_U = B_1 y^{M_1}, S_D = B_2 y^{M_2} \]  \hspace{1cm} (33)

Where \( y \) denotes the flow depth, \( I \) and \( Q \) denote the flow rates of upstream and downstream sections, respectively, \( S_U \) and \( S_D \) denote the storages related to the depths at the upstream and downstream sections, respectively, \( A_1, N_1 \) and \( A_2, N_2 \) denote the depth-discharge characteristics of the upstream and downstream sections, respectively, and \( B_1, M_1 \) and \( B_2, M_2 \) denote the mean depth-storage characteristics of the upstream and downstream sections of the reach, respectively.

By combining Eqs. (32) and (33) and eliminating flow depth, \( S_U \) and \( S_D \) are:

\[ S_U = B_1 \left( \frac{I}{A_1} \right)^{M_1/N_1}, \quad S_D = B_2 \left( \frac{Q}{A_2} \right)^{M_2/N_2} \]  \hspace{1cm} (34)

As expressed by Easa (2013), the storage in a channel, \( S \), can considered as a power function of the weighted storage:

\[ S = [\chi S_U + (1 - \chi)S_D]^m \]  \hspace{1cm} (35)

Where \( \chi \) denotes the weighting parameter and \( m \) denotes the exponent parameter.

Finally, by substituting \( S_U \) and \( S_D \) from Eq. (34) into Eq. (35), the new and general form of the storage equation of Muskingum procedure is:

\[ S = [\alpha_1 I^{p_1} + \alpha_2 Q^{p_2}]^m \]  \hspace{1cm} (36)

Where \( \alpha_1 = (\chi B_1) / (A_1)^{M_1/N_1}, p_1 = M_1/N_1, \alpha_2 = (1 - \chi) B_2 / (A_2)^{M_2/N_2} \) and \( p_2 = M_2/N_2 \).

It is notable that Eq. (36) and Eq. (26) have similar structure. However, in contrast to Eq. (26), Eq. (36) was derived based on the physical and mathematical concepts of flooding simulation.

4. Discussion of Performance Evaluation Criteria

In the original paper, four performance evaluation criteria were adopted in order to the comparison between the results of different calibration methods of the flood routing procedure. These criteria are (1) the sum of the square of the deviation between the routed and observed outflows (SSQ) as accuracy criterion; (2) the absolute value of the deviations of peak of routed and observed outflows (DPO) as magnitude of peak criterion; (3) The mean absolutely relative error between the routed and observed outflows (MARE) as mean absolutely relative error criterion; and (4) the Nash–Sutcliff criterion of the variance explained (\( \eta^2 \)) as closeness of shape and size of the hydrograph criterion. Based on the results of the original paper, the Excel solver using the discrete-time method of Tung (1985) shows good results for all of the criteria.

The discusser used the sum of the square of the deviations between the observed and computed storage rates (SSSR) as a measure using two different formula for calculation of the storage rates of observed values and computed values, respectively, as:

\[ S_{\text{observed}} = \frac{1}{2} (Q_{t+1} - Q_{t+1}) \]  \hspace{1cm} (37)

\[ S_{\text{computed}} = \frac{1}{2} (S_{t+1} - (1 - \lambda) S_{t+1}) \]  \hspace{1cm} (38)

For true comparison, both \( S_{\text{observed}} \) and \( S_{\text{computed}} \) must be calculate with a same equation using either Eq. (37) or Eq. (38).

Furthermore, based on the Eq. (14) of the discrete-time method of Tung (1985) in the original paper, \( S_{\text{computed}} \) was calculated as:

\[ S_{\text{computed}} = \frac{1}{2} \left[ \frac{1}{1 - \lambda} \left( \frac{S_M}{K} \right)^{1/m} + \left( \frac{1}{1 - \lambda} \right) b \right] \]  \hspace{1cm} (39)

Therefore, the use of Eq. (38) for calculation of the storage rates of computed values of the discrete-time method is questionable.

On the other hand, SSSR has not physically concept in the flood routing procedure. Water resources engineers use the flood routing procedure to determine the changes of discharge information of the flood event along a river (Chow, 1959; Perumal and Sahoo, 2007; Akbari and Barati, 2012; Akbari et al., 2012).

Finally, it can be said that SSSR which originally presented by the discusser is not a standard performance evaluation criteria.

5. Discussion of Numerical Solution Method

In general, several numerical solution methods such as explicit and implicit Euler methods, Heun’s method and Runge–Kutta methods (Butcher, 2008) can be used to calculate the storage time variation of the Muskingum procedure. However, the accuracy of the methods is important in the simulation procedure.

Traditionally, researchers used Tung’s method which based on the Euler method using the concept of state variable modeling (e.g., Tung, 1985; Luo and Xie, 2010; Xu et al., 2012; Easa, 2012; Orouji et al., 2013; Easa, 2013). In this procedure, the Euler method was used to calculate the time variation of the storage. The discusser claimed that the use of \( I_i \) instead of \( I \) in the following routing equation is incorrect from the numerical