The effects of non-uniform flow velocity on vibrations of single-walled carbon nanotube conveying fluid

Moslem Sadeghi-Goughari¹ and Mohammad Hosseini²*¹

¹Department of Mechanical Engineering, College of Technology of Sirjan, Shahid Bahonar University of Kerman, 76169-14111 Kerman, Iran
²Department of Mechanical Engineering, Sirjan University of Technology, 78137-33385 Sirjan, Iran

(Manuscript Received April 9, 2014; Revised September 18, 2014; Accepted November 2, 2014)

Abstract

The vibrational behavior of a viscous nanoflow-conveying single-walled carbon nanotube (SWCNT) was investigated. The non-uniformity of the flow velocity distribution caused by the viscosity of fluid and the small-size effects on the flow field was considered. Euler-Bernoulli beam model was used to investigate flow-induced vibration of the nanotube, while the non-uniformity of the flow velocity and the small-size effects of the flow field were formulated through Knudsen number (Kn), as a discriminant parameter. For laminar flow in a circular nanotube, the momentum correction factor was developed as a function of Kn. For Kn = 0 (continuum flow), the momentum correction factor was found to be 1.33, which decreases by the increase in Kn may even reach near 1 for the transition flow regime. We observed that for passage of viscous flow through a nanotube with the non-uniform flow velocity, the critical continuum flow velocity for divergence decreased considerably as opposed to those for the uniform flow velocity, while by increasing Kn, the difference between the uniform and non-uniform flow models may be reduced. In the solution part, the differential transformation method (DTM) was used to solve the governing differential equations of motion.

Keywords: Single-walled carbon nanotube; Laminar flow; Non-uniform flow velocity; Slip boundary conditions; Knudsen number; Differential transformation method

1. Introduction

Carbon nanotubes (CNTs) have unique electronic, chemical, thermal, fluid-transport and gas storage properties [1]. They have been recognized as the nanostructures for enormous applications ranging from nanomechanical systems to nanobiological devices. In the nanomechanical systems, CNTs can be used for many applications including mechanical energy harvesting, mechanosensing, nanotube for conveying fluid and nanocontainers for gas storage [2]. In drug delivery systems, CNTs are used to act as nanochannels for delivering drug into target cells [3]. Significant improvement in the efficiency of these nanobiological devices and nanomechanical systems is related to our deep understanding of their dynamics [4].

The influence of internal moving fluid on overall mechanical behavior of CNTs is a significant and challenging research topic [5]. As a result, the dynamic analysis of CNTs conveying fluid has become a common subject in recent years.

Yoon et al. [5] investigated the influence of flow velocity on the vibrational behavior of CNTs conveying fluid and indicated that the internal moving fluid could substantially affect the vibrational frequencies and stability of CNT. Other researches dealt with additional factors that affected the stability of single-walled (SWCNT) and multi-walled CNTs (MWCNTs). For example, Ali-Akbari et al. [6] studied the instability condition of large diameter SWCNTs conveying fluid by using the molecular mechanics method. The nonlinear vibration and flow-induced instability of the double-walled boron nitride nanotubes embedded in linear viscoelastic Pasternak medium, based on the modified couple stress theory, with electromechanical coupling condition was investigated by Ghorbanpour Arani et al. [7]. The influence of the tangential distributed force on the structural instability of the CNTs studied by Kazemi-Lari et al. [8]. The fluid-induced vibration and flutter instability of a cantilever MWCNT conveying fluid investigated by Yun et al. [9].

The mechanical behavior of fluid flow in nano-size is different from those of macro/meso scales. Surface-dominant effects, small size effects and low Reynolds number effects are the most significant differences among others [10]. The small size effects on the flow field can be considered through Knudsen number (Kn). Kn, the ratio of mean free path to a characteristic length of problem geometry, is utilized as a discriminant parameter for identification of flow regime. In a nano-scale fluid structure interaction (FSI) problem, Kn may
be larger than $10^3$ and, consequently, the assumption of no-slip boundary conditions between the flow and nanotube walls is invalid [11]. Therefore, the Navier-Stokes continuum equations should be modified for the slip flow regime.

Besides, several studies on the influence of $Kn$ on the dynamics of nanotubes conveying fluid have been reported. By considering the small-size effects on the flow field, Rashidi et al. [11] proposed a novel model for single-mode coupled vibrations of CNTs conveying fluid. They formulated the small-size effects on bulk viscosity and the slip boundary conditions of nanoflows through $Kn$ and devised a dimensionless parameter, called velocity correction factor ($VCF$), in order to modify the FSI governing equations. It was found that for passage of gas through a nanotube, ignoring the small-size effects on flow field in a nano-scale FSI problem might generate erroneous results. Mirramezan and Mirdamadi [12] studied higher-modes stability (flutter) and dynamics of nanotube conveying gas flow. They revealed that coupled-mode flutter for a $Kn$ higher than zero might occur much sooner than for $Kn$ equal to zero. Kaviani and Mirdamadi [13] considered the small size effects of viscosity of fluid flow and modified the parameter $VCF$ introduced by Rashidi et al. [11]. They revealed that this new formulation of $VCF$ generated different results as compared to the case where nano-sized fluid viscosity had been ignored in slip boundary conditions. Nonlinear free vibration and instability of viscous fluid-conveying double-walled carbon nanocomes studied via DQM by Ghorbapour Arani et al. [14]. In their work the small-size effects on bulk viscosity and slip boundary conditions of nanoflow through $Kn$ were considered. The effects of nano-size of both fluid flow and nanotube on vibration of a SWCNT conveying nanoflow embedded in biological soft tissue were considered by Hosseini et al. [15].

In all of the previous studies on the influence of small size effects on the dynamics of nanotube conveying nanoflow, the flow velocity was considered to be uniform. The assumption of uniform flow velocity may be justified for a fluid flow at high Reynolds numbers, while the velocity profile is nearly uniform and this assumption is less valid at low Reynolds numbers. To the authors’ knowledge, none of the mentioned studies on the influences of small size effects considered an analytical approach to take into account the non-uniformity for flow velocity distribution in fluid-conveying nanotubes caused by the viscosity of real fluids.

As for the literature published so far, several methods have been used for solving the FSI problems. In this research, the differential transformation method (DTM) is used to investigate the free vibration of SWCNT conveying nanoflow. In fact, DTM is a semi-analytical method for solving ordinary and partial differential equations which is based on Taylor’s series expansion. Taking advantage of some efficient transformation rules, the DTM is applied to convert the governing differential equations of motion into a set of algebraic equations. The DTM was first proposed by Zhou [16] for solving linear and non-linear initial value problems in electrical circuit analysis. Ni et al. [17] showed that the DTM has high precision and computational efficiency in acquiring a prognosis of the vibrational characteristics of a pipe conveying fluid.

Our main objective is to present a model for the coupled vibrations of nanotubes conveying fluid, considering the non-uniformity of the flow velocity caused by the fluid viscosity. We have presented a modified model that incorporates the non-uniformity of the flow velocity and the slip boundary conditions on the nanotube walls for acquiring a more exact prognosis of the vibrational characteristics of a SWCNT conveying nanoflow. Furthermore, the differential transformation method has been utilized to solve the FSI governing partial differential equations.

2. The FSI governing equations

The convectional governing equation of motion for vibrations of a pipe conveying fluid, ignoring gravity and externally imposed tension and pressurization effects, can be expressed as [18]:

$$\frac{EI}{\partial x^4} + MU^2 \frac{\partial^2 W}{\partial x^2} + 2MU \frac{\partial^2 \tilde{W}}{\partial x \partial T} + (M + m) \frac{\partial^2 \tilde{W}}{\partial T^2} = 0, \quad (1)$$

where $EI$ is the flexural rigidity of the nanotube, $M$ is the fluid mass per unit length, $m$ is the mass of nanotube per unit length, $U$ is the mean flow velocity and $W = W(X,T)$ is the flexural displacement of the nanotube wall at the axial coordinate $X$ and time $T$.

The non-dimensional form of Eq. (1) can be written as:

$$\frac{\partial^4 W}{\partial x^4} + \frac{u^2}{\beta^2} \frac{\partial^2 W}{\partial x^2} + 2u \sqrt{\beta} \frac{\partial^2 \tilde{W}}{\partial x \partial T} + \frac{\partial^2 \tilde{W}}{\partial T^2} = 0, \quad (2)$$

by definition of the dimensionless parameters and variables:

$$w = \frac{W}{L}, \quad x = \frac{X}{L}, \quad t = \frac{T}{L^2}, \quad \beta = \frac{M}{M + m}, \quad u = \left( \frac{M}{EI} \right)^{\frac{1}{2}} UL \quad (3)$$

where $L$ is the nanotube length, $\beta$ is the mass ratio and $u$ is the dimensionless fluid flow velocity.

The FSI governing equations of motion for the nanotube conveying fluid basically derived by two major assumptions, no-slip boundary conditions between the flow and nanotube walls surface and using the ideal fluid flow (uniform flow) model for flow in nanotube. With due attention to the flow behavior at nano-scale such as, surface dominant effects, small size effects and low Reynolds number effects, these conditions are no longer valid and the FSI governing equations should be modified.

2.1 Modeling slip boundary conditions

For considering the nanoflow slip boundary conditions, the