Robust Adaptive Dynamic Surface Control of Uncertain Nonlinear Systems

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Abstract: This paper studies the robust adaptive dynamic surface control of a class of nonlinear systems with unmatched uncertainties. The unmatched uncertainties consist of not only the linearly parameterized terms but also the nonlinearly parameterized terms. The bound of each nonlinearly parameterized uncertainty term is supposed to be expressed by a known nonnegative function multiplied by a constant called bound parameter. According to whether the bound parameters are known or not, two different kinds of robust adaptive dynamic surface control algorithms are proposed. It is proved that in each case all the states of the closed-loop system are kept uniformly ultimately bounded, and the output is driven to track a feasible desired output trajectory with an arbitrarily small error. An example is also employed to indicate the effect of the proposed methods.

Keywords: Dynamic surface control, nonlinear system, robust adaptive control, unmatched uncertainty.

1. INTRODUCTION

Feedback control of uncertain nonlinear systems by robust or adaptive control techniques is a problem of paramount importance in the field of control and has received considerable attention, since nonlinearities and uncertainties exist inherently in many real systems [1, 2]. A powerful tool for solving such a problem is the backstepping technique. This control design methodology was first proposed for parametric strict-feedback systems with linearly parameterized uncertainties [3, 4], and then extended to deal with nonlinearly parameterized uncertainties [5, 6]. After nearly twenty years of development, it has become one of the most popular design methods for a large class of nonlinear systems with uncertainties, especially with unmatched uncertainties [7, 8].

However, the backstepping technique suffers from the problem of explosion of complexity arising from the repeated differentiations of the virtual controls. As a result, the complexity of controller grows drastically as the order of the system increases. In addition, it requires certain system functions to be $C^n$. To avoid these problems, the dynamic surface control (DSC) technique was proposed in [9] and [10] for nonlinear systems with unmatched uncertainties by introducing a first-order filtering of the synthetic input at each step of the traditional backstepping approach. So far, this control method has been implemented successfully in many practical applications such as friction compensation [11], anti-lock brake system [12], formation control [13], magnetic levitation system [14], underactuated mechanical system [15], and so on. More details about how to synthesize the design parameters in DSC can be found in [16]. In [17] the DSC method was applied to a class of nonlinear systems with linearly parameterized uncertainties. However, if this linear parameterization assumption is not satisfied, the adaptive dynamic surface control design will become more difficult and challenging. For this problem, some results have been obtained combining with fuzzy control [18] or neural network control [19]. For more developments about DSC incorporating with intelligent control, the interested readers can see [20] and the references therein.

In the current paper, we further consider the output tracking problem of nonlinear systems with unmatched uncertainties, which include both of the linearly parameterized parts and the nonlinearly parameterized parts. Specifically, we consider a class of single-input-single-output uncertain nonlinear systems of the form

$$\begin{align*}
\dot{x}_i &= x_{i+1} + \theta_i f_i(x_i) + \delta_i(x), i = 1, 2, \ldots, n-1 \\
\dot{x}_n &= u + \theta_n f_n(x) + \delta_n(x) \\
y &= x_1,
\end{align*}$$

(1)

where $x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the state, the input and the output of the system, respectively; for $i = 1, 2, \ldots, n$, $\theta_i$ are all unknown constants, $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T$, $f_i(\bar{x}_i) : \mathbb{R}^i \to \mathbb{R}$ are $C^4$ functions, $\delta_i(x) : \mathbb{R}^n \to \mathbb{R}$ are continuous functions, the terms $\theta_i f_i(\bar{x}_i)$ represent the linearly parameterized uncertainties, and the terms $\delta_i(x)$ represent the nonlinear parameterized uncertainties and are supposed to satisfy the following assumption.

**Assumption 1:** There exist a set of possibly unknown constants $\rho_i \geq 0$, $i = 1, 2, \ldots, n$, which are called as...
bound parameters, and a set of known nonnegative $C^1$ functions $\phi_i(\bar{x})$ such that
\[
|\delta_i(x)| \leq \rho_i \phi_i(\bar{x}).
\] (2)

The main objective of this paper is to develop the robust adaptive dynamic surface control (RADSC) methods for system (1) under Assumption 1 to make the output track a desired output trajectory $x_{id}$ and keep the other states bounded. Here, the feasible desired output trajectories are supposed to satisfy the assumption below.

**Assumption 2:** The desired output trajectory $x_{id}$ is a sufficiently smooth function and available, and satisfies
\[
[x_{id} \ x_{\bar{id}} \ x_{\bar{id}}]^T B_0 = \{(z_1 \ z_2 \ z_3)^T : z_1^2 + z_2^2 + z_3^2 \leq r_0^2\},
\]
where $r_0$ is a known constant.

In this paper, two different kinds of robust adaptive dynamic surface controllers are proposed according to whether the bound parameters $\rho_i$ are known or not. It is proved that by using the proposed robust adaptive controllers with proper design parameters, the output of the closed-loop system can track the desired output trajectory with arbitrary small tracking error, and the other states are kept bounded simultaneously. As a result, a systematic design procedure is established to handle the output regulation problem of a large class of nonlinear systems with unmatched uncertainties. Throughout this paper, $|\cdot|$ represents the absolute value of a real number, $\|\cdot\|$ represents the Euclidean norm of a vector or the spectral norm of a matrix. For convenience, we denote a vector $[v_1 \ v_2 \ \cdots \ v_i]^T$ by $\bar{v}_i$.

**2. ROBUST ADAPTIVE DYNAMIC SURFACE CONTROL DESIGN**

This section first presents the robust adaptive dynamic surface control for system (1) under Assumption 1 when the bound parameters are known, and then gives the result when the bound parameters are unknown.

2.1. Control design for known bound parameters

2.1.1 Control algorithm

When the bound parameters $\rho_i, i = 1, 2, \cdots, n$ are known, the RADSC algorithm for system (1) is described as follows.

**Algorithm 1:**

**Step** $1 (1 \leq i \leq n - 1)$:

\[
S_i = x_i - x_{id},
\]
\[
\bar{x}_{id} = \bar{x}_{id} - K_i S_i - \hat{\theta}_i f_i(x_i) - \rho_i^2 \theta_i^2 (x_i) S_i,
\]
\[
\bar{\tau}_{id} \bar{x}_{id} + x_{id} = \bar{x}_{id}, x_{id}(0) = \bar{x}_{id}(0)
\]

**Step** $n$:

\[
S_n = x_n - x_{ad},
\]
\[
u = \bar{\delta}_n - K_n S_n - \hat{\theta}_n f_n(x) - \rho_n^2 \theta_n^2 (x) S_n,
\]

where the design parameters $K_1, \cdots, K_n$ are called as surface gains; $\tau_2, \cdots, \tau_n$ are called as filter time constants; and $\hat{\theta}_i$ is the estimate of the unknown parameter $\theta_i$ and satisfies the following update law
\[
\dot{\hat{\theta}}_i = \lambda_i \left[ S_i f_i(x) + \pi_i (\theta_{i0} - \hat{\theta}) \right], i = 1, 2, \cdots, n,
\]

where $\lambda_i$, $\pi_i$ and $\theta_{i0}$ are all design parameters, and $\theta_{i0}$ is the predictive value of the unknown parameter $\theta_i$.

2.1.2 Stability analysis

Define the boundary layer errors as
\[
y_i = x_{id} - \bar{x}_i, i = 2, \cdots, n,
\]
and the estimate errors as
\[
\hat{\theta}_i = \theta_i - \hat{\theta}_i, i = 1, 2, \cdots, n.
\]

Then the closed-loop dynamics can be expressed in terms of the surfaces ($S_i$), the boundary layer errors ($y_i$) and the estimate errors ($\hat{\theta}_i$). The dynamics of the surfaces are expressed as, for $i = 1, 2, \cdots, n - 1$,
\[
\dot{S}_i = \dot{x}_i - \dot{\bar{x}}_{id},
\]
\[
= x_{i+1} + \bar{\theta}_i f_i(x_i) + \delta_i(x) - \bar{x}_{id},
\]
\[
= S_{i+1} + y_{i+1} + \bar{S}_{i+1} + \bar{\theta}_i f_i(x_i) + \delta_i(x) - \bar{x}_{id},
\]
\[
= S_{i+1} + y_{i+1} + \bar{K}_i S_i + \bar{\theta}_i f_i(x_i) - \rho_i^2 \theta_i^2 (x_i) S_i + \delta_i(x),
\]

and
\[
\dot{S}_n = \dot{x}_n - \dot{\bar{x}}_{ad},
\]
\[
= u + \bar{\theta}_n f_n(x) + \delta_n(x) - \bar{x}_{ad},
\]
\[
= -K_n S_n + \bar{\theta}_n f_n(x) - \rho_n^2 \theta_n^2 (x) S_n + \delta_n(x).
\]

The dynamics of the boundary layer errors are expressed as, for $i = 2, \cdots, n$,
\[
y_i = -\frac{1}{\tau_i} y_i - \bar{x}_i.
\]

The dynamics of the estimate errors are expressed as, for $i = 1, \cdots, n$,
\[
\dot{\hat{\theta}}_i = -\dot{\hat{\theta}}_i = -\lambda_i \left[ S_i f_i(x_i) + \pi_i (\theta_{i0} - \hat{\theta}_i) \right].
\]

Let
\[
V = \sum_{i=1}^{n} V_{ii} + \sum_{i=2}^{n} V_{ij} + \sum_{i=1}^{n} V_{i0},
\]

where
\[
V_{ii} = \frac{1}{2} S_i^2, \quad V_{ij} = \frac{1}{2} y_i^2, \quad V_{i0} = \frac{1}{2} \bar{\theta}_i^2,
\]

then one has