**H_/H_/∞** Sensor Fault Detection and Isolation in Linear Time-Invariant Systems

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**Abstract:** This paper addresses sensor fault detection and isolation problems for continuous- and discrete-time linear time-invariant systems. To that end, we employ a bank consisting of the same number of observers as there are sensors. Both the observer gain and the residual gain are considered. Unlike earlier work, the design conditions with **H_/H_/∞** performance are derived in terms of linear, rather than nonlinear, matrix inequalities. An illustrative example is provided to show the effectiveness of the proposed methodology.

**Keywords:** Fault detection and isolation, linear matrix inequalities, observer bank, **H_/H_/∞** performance.

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1. INTRODUCTION

Modern systems have the potential of sensor faults in the presence of disturbance inflow due to their complex structure. Sensor faults are now considered to be the main source of malfunctions in the whole system. In order to protect a system from malfunctions, sensor faults must be clearly detected (i.e., a fault is recognized) and isolated (i.e., a faulty sensor is determined) subject to the sensor disturbance [1]. Several approaches to fault detection and isolation (FDI) have been pursued so far, among which the model-based observer scheme has attracted a good deal of attentions [1-8].

In single-output systems, detecting a fault via a specific diagnostic signal called a residual through an observer suffices to isolate it. However, this is not the case in multi-output systems. Further information beyond the fault detection—the location of a faulty sensor—is required. To that end, the residual must be generated so that each entry is sensitive only to the fault occurring on a particular sensor. This can be achieved by building a non-interactive (diagonal) map from the fault to the residual, which however is quite complicated to be designed in practice even for linear time-invariant (LTI) systems [9,10]. A simple but effective alternative is to use a bank of as many observers as there are sensors [3]. The residual generated by each observer in the bank should be as sensitive to the faults on all sensors except one and robust against the disturbance, as possible.

Then the fault can be isolated based on a suitable voting scheme [1].

There have been several studies on the observer-bank-based FDI with different viewpoints. An FDI method via an observer bank and its application to load-frequency control is studied in [3]. However neither any explicit performance criterion nor residual gain is considered therein. In [1], a robust FDI for a robot manipulator is developed where residual sensitivity to fault is considered in the **H_/∞** sense. Paper [11] proposes an FDI technique for uncertain LTI systems. Herein, fault isolation is formulated by the **H_/∞** model-matching problem, where the isolation performance may be dependent on the choice of the reference model. The (diagonal) identity matrix as a best reference model would yields the infeasibility of the associated design condition.

Yet another straightforward strategy in designing the observer bank for the FDI may be the **H_/H_/∞** framework where the disturbance attenuation is formulated by **H_/∞** performance and the fault sensitivity is characterized by the **H_/∞** performance. In [12,13], the **H_/∞** performance together with residual gain are introduced for fault detection. However, fault isolation is not fully investigated. Moreover, it is observed that although their design conditions are elegant, they are not linear matrix inequalities (LMIs) even if the target systems are LTI. The reason seems to be that the product of the residual gain and its transposition that arises in the **H_/∞** performance imposition may not be decomposed for convexification of the derived nonlinear matrix inequality condition. Consequently, their residual gains must be found in a symmetric positive definite form or a triangular form through additional manipulation such as the matrix square root or a Cholesky factorization.

Motivated by the observations above, this paper develops design conditions of the **H_/H_/∞** FDI observer banks for continuous- and discrete-time LTI systems. The bank is composed of the equal number of Luenberger-type observers as there are sensors. Both the observer gain and the residual gain (that is not necessarily symmetric or triangular) are taken into
account. Sufficient conditions to find the gains are formulated in the LMI format. An example is included to visualize the theoretical analysis and design.

We follow standard notations: $A = A^T < 0$ is a negative definite matrix. $||x||$ stands for a Euclidean norm while $(||x||_2)$ means the $L_2$ norm. $*$ denotes a transposed element in a symmetric position. Ellipsis means the $l_1$ $(L_2)$ norm. Without loss of generality, it is assumed that $A$ is Hurwitz and $(A, C)$ is observable. We adopt a bank of $m$ observers shown in Fig. 1, in which the $p$ th observer takes the following Luenberger’s form:

$$\begin{align*}
\dot{x}^p &= Ax + Dw + f^p, \\
y^p &= Cx + D w^p, \\
\hat{x}^p &= A \hat{x}^p - L^p (y^p - \hat{y}^p) \\
\hat{y}^p &= C \hat{x}^p,
\end{align*}$$

where $x \in \mathbb{R}^n$ is the state, $y \in \mathbb{R}^m$ is the output, $w \in \mathbb{R}^q$ is the sensor disturbance, and $f \in \mathbb{R}^m$ is the sensor fault. Without loss of generality, it is assumed that $A$ is Hurwitz and $(A, C)$ is observable. We adopt a bank of $m$ observers shown in Fig. 1, in which the $p$ th observer is isolated if $y^p - \hat{y}^p$ is fed back to (2). It means that the $r^p$ is completely insensitive to the fault on the $p$ th sensor.

Let $e^p = x - \hat{x}^p$. The residual dynamics governed by (2) is written as

$$\begin{align*}
\dot{e}^p &= (A + L^C C^p) e^p + L^p D w + L^p f^p \\
r^p &= H^p C e^p + H^p D w + H^p f^p.
\end{align*}$$

Based on these $m$ residuals we say that a fault is detected when

$$J^p(t) \geq J^p_{th}(t)$$

for some $p \in \mathcal{I}_M$ and some $t \in \mathbb{R}_{>0}$, where

$$J^p(t) = \frac{1}{T_W} \int_{t-T_W}^t \|r^p(\tau)\|^2 d\tau$$

is the $p$ th residual evaluation function and

$$J^p_{th}(t) = \sup_{w \in L_2, f \in L_2} \frac{1}{T_W} \int_{t-T_W}^t \|r^p(\tau)\|^2 d\tau,$$

is the associated threshold, where $T_W \in \mathbb{R}_{>0}$ is the constant time window. We also say that a fault on the $q$ th sensor is isolated if

$$\begin{align*}
J^q(t) < J^q_{th}(t) \\
J^q(t) \geq J^q_{th}(t)
\end{align*}$$

for all $p \in \mathcal{I}_M \setminus \{q\}$ and some $t \in \mathbb{R}_{>0}$.

For good FDI, (2) is desired to be designed so that the sensitivity from $r^p$ to $r^p$ is encouraged and the sensitivity from $w$ to $r^p$ is suppressed. We recall the following definition.

**Definition 1:** For a map from $f^p$ to $r^p$ in (3), the $H_\infty$ performance is defined by [13]

$$\|Q_p\|_{\infty} = \inf_{f^p \in L_2} \sup_{t \in \mathbb{R}_{\geq 0}} \frac{\|r^p(t)\|}{\|f^p(t)\|} = \inf_{f^p \in L_2} \frac{1}{T_W} \int_{t-T_W}^t \frac{\|r^p(\tau)\|^2}{\|f^p(\tau)\|^2} d\tau.$$